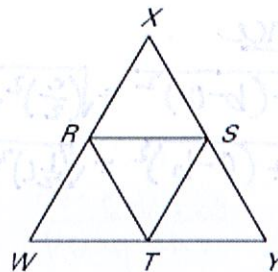


Use $\triangle WXY$, where R, S, and T are midpoints of the sides.

- $\overline{RS} \parallel \overline{WY}$
- $\overline{ST} \parallel \overline{WX}$
- If $TY = 4$, then $RS = 4$
- If $RT = 7$, then $XY = 14$



Use the diagram of $\triangle ABC$ where D, E, and F are the midpoints of the sides. $3x+36$

- If $FE = 6.5x - 1$ and $AB = 3x + 20$, then $AB = 26.6$

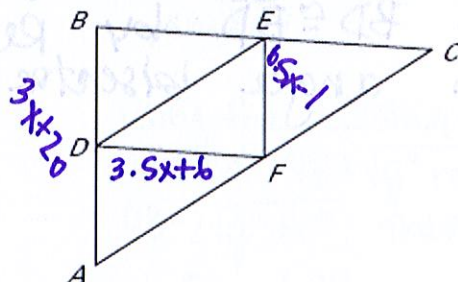
$$\begin{aligned} 2(6.5x - 1) &= 3x + 20 \\ 13x - 2 &= 3x + 20 \\ 10x &= 22 \end{aligned}$$

$$x = 2.2$$

- If $DF = 3.5x + 6$ and $BC = 3x + 36$, then $DF = 27$

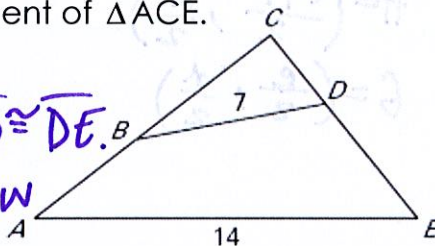
$$\begin{aligned} 2(3.5x + 6) &= 3x + 36 \\ 7x + 12 &= 3x + 36 \\ 4x &= 24 \\ x &= 6 \end{aligned}$$

$$\begin{aligned} DF &= 3.5(6) + 6 \\ DF &= 27 \end{aligned}$$



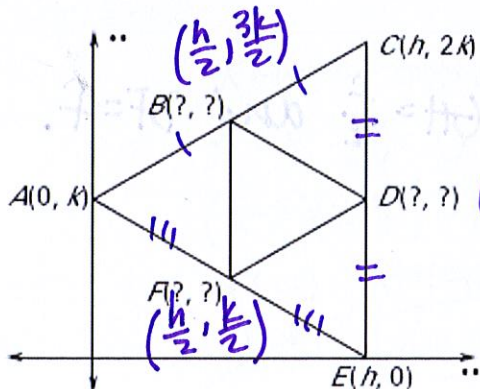
- Explain why $\overline{BD}$ is not a midsegment of $\triangle ACE$.

You don't know $\overline{AB} \cong \overline{BC}$ and $\overline{CD} \cong \overline{DE}$.
Also, you don't know $\overline{BD} \parallel \overline{AE}$.



Find the unknown coordinates of the points in the figure.

8.

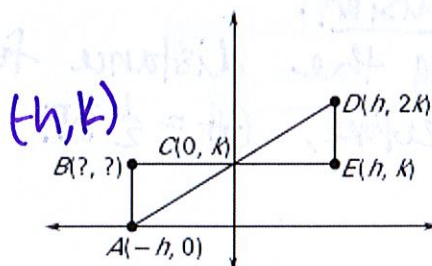


B, D, & F are midpoints!

$$(h, k)$$

Midpoint formula!

$$\triangle ABC \cong \triangle DEC$$



10. Coordinate proof

Given: $\overline{DB}$ bisects $\angle ABC$

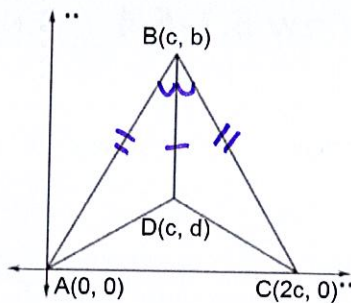
Prove: $\triangle ABD \cong \triangle CBD$

WORK: distance

$$AB = \sqrt{(c-0)^2 + (b-0)^2}$$

$$AB = \sqrt{c^2 + b^2}$$

$$BC = \sqrt{(2c-c)^2 + (0-b)^2} = \sqrt{c^2 + (-b)^2} = \sqrt{c^2 + b^2}$$



conclusion:

using the distance formula, I found AB and BC both have a length of $\sqrt{c^2 + b^2}$, making them congruent. since $\overline{BD} \cong \overline{BD}$ by the reflexive property and $\angle ABD \cong \angle CBD$ by definition of angle bisector. Therefore, $\triangle ABD \cong \triangle CBD$ by SAS.

11. Coordinate proof

Given: Coordinates of $\triangle DEF$

G is the midpoint of $\overline{DE}$

H is the midpoint of $\overline{EF}$

Midpoint:
 $H = \left(\frac{e+f}{2}, \frac{d}{2} \right)$

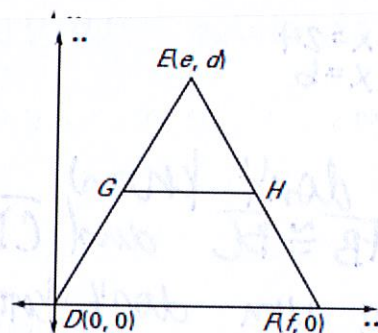
$$G = \left(\frac{e}{2}, \frac{d}{2} \right)$$

Prove: $GH = \frac{1}{2}DF$

WORK: distance

$$GH = \sqrt{\left(\frac{e+f}{2} - \frac{e}{2} \right)^2 + \left(\frac{d}{2} - \frac{d}{2} \right)^2} = \sqrt{\left(\frac{f}{2} \right)^2 + 0^2} = \frac{f}{2}$$

$$DF = \sqrt{(f-0)^2 + (0-0)^2} = \sqrt{f^2} = f$$



conclusion:

Using the distance formula, I found $GH = \frac{f}{2}$ and $DF = f$. therefore, $GH = \frac{1}{2}DF$.

12. Coordinate Proof

Given: $\overline{FE}$ is a midsegment

Prove: $\overline{FE} \parallel \overline{OB}$ and $FE = \frac{1}{2} OB$

(use midpoint to find coordinates of F and E!)

Midpoint:

$$F = \left(\frac{2p+0}{2}, \frac{0+0}{2} \right) = (p, 0) \quad E = \left(\frac{2p+2q}{2}, \frac{2r+0}{2} \right) = (p+q, r)$$

Slope:

$$m_{FE} = \frac{r-0}{p+q-p} = \frac{r}{q}$$

$$m_{OB} = \frac{2r-0}{2q-0} = \frac{2r}{2q} = \frac{r}{q}$$

Distance:

$$FE = \sqrt{(p+q-p)^2 + (r-0)^2}$$

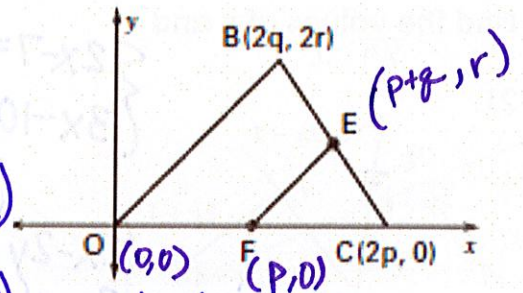
$$FE = \sqrt{q^2 + r^2}$$

$$OB = \sqrt{(2q-0)^2 + (2r-0)^2}$$

$$OB = \sqrt{4q^2 + 4r^2}$$

$$OB = \sqrt{4(q^2 + r^2)}$$

$$OB = 2\sqrt{q^2 + r^2}$$



conclusion:

Using the slope formula, I found $\overline{FE}$ and $\overline{OB}$ both have a slope of $\frac{r}{q}$, making them parallel.

using the distance formula, I found $FE = \sqrt{q^2 + r^2}$ and $OB = 2\sqrt{q^2 + r^2}$. Therefore $FE = \frac{1}{2} OB$.

Use the diagram. $\overline{EH}$ is perpendicular bisector of $\overline{DF}$. Find the indicated measure.

13. Find EF **44**

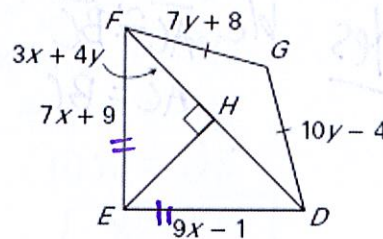
14. Find DE **44**

15. Find FG **36**

16. Find DG **36**

17. Find FH **31**

18. Find DF **62**



$$7x + 9 = 9x - 1$$

$$10 = 2x$$

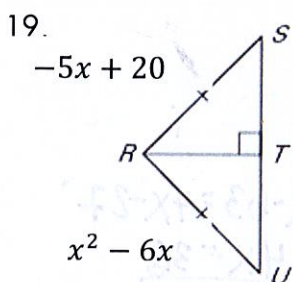
$$5 = x$$

$$7y + 8 = 10y - 4$$

$$12 = 3y$$

$$4 = y$$

Find the value of x.



$$x^2 - 6x = -5x + 20$$

$$x^2 - x - 20 = 0$$

$$(x-5)(x+4) = 0$$

$$x-5 = 0$$

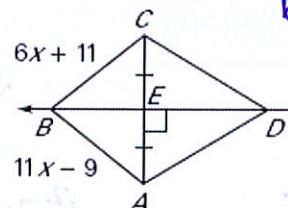
$$x+4 = 0$$

$$x = 5$$

$$x = -4$$

the negative works!!

20.



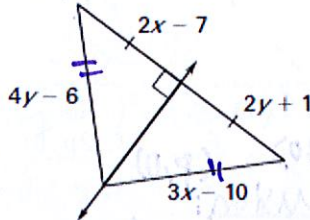
$$6x + 11 = 11x - 9$$

$$20 = 5x$$

$$4 = x$$

Find the values of x and y .

21.



$$\begin{cases} 2x-7=2y+1 \\ 3x-10=4y-6 \end{cases}$$

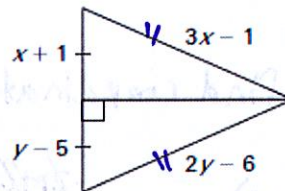
$$\begin{aligned} (2x-2y=8) \times -2 \\ 3x-4y=4 \end{aligned}$$

$$\begin{aligned} -4x+4y &= -16 \\ + 3x-4y &= 4 \\ \hline -x &= -12 \end{aligned}$$

$$\boxed{y=8}$$

$$\boxed{x=12}$$

22.



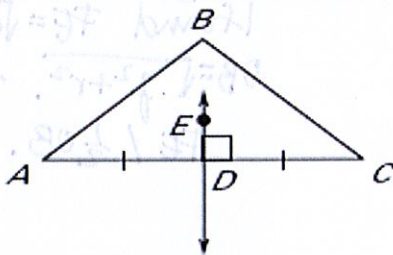
$$\begin{cases} 3x-1=2y-6 \\ x+1=y-5 \end{cases}$$

$$\begin{aligned} 3x-2y &= -5 \\ (x-y=-6) \times -2 \end{aligned}$$

$$\begin{aligned} 3x-2y &= -5 \\ -2x+2y &= 12 \\ \hline x &= 7 \end{aligned}$$

$$\boxed{\begin{matrix} x=7 \\ y=13 \end{matrix}}$$

23. Explain why the conclusion is not correct given the information in the diagram.

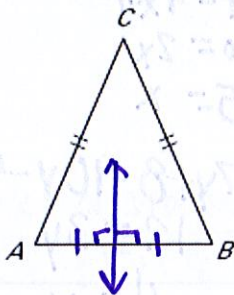


$\overline{DE}$ will pass through B .

We do not know $AB=BC$.
 B must be equidistant from both endpoints.

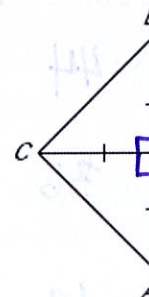
Tell whether the information in the diagram allows you to conclude that C is on the perpendicular bisector of $\overline{AB}$.

24.



Yes b/c $\overline{AC} \cong \overline{BC}$
 $AC=BC$

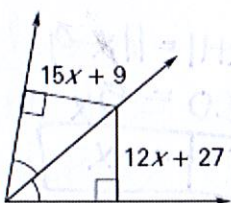
25.



No. we do not know $BC=AC$.

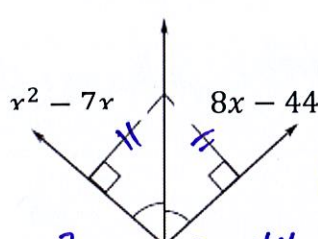
Find the value of x .

26.



$$\begin{aligned} 15x+9 &= 12x+27 \\ 3x &= 18 \\ \boxed{x=6} \end{aligned}$$

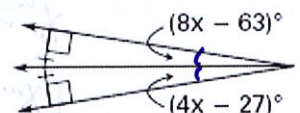
27.



$$\begin{aligned} x^2-7x &= 8x-44 \\ x^2-15x+44 &= 0 \\ (x-11)(x-4) &= 0 \\ x-11 &= 0 \quad x-4 &= 0 \end{aligned}$$

doesn't work when plugging back in

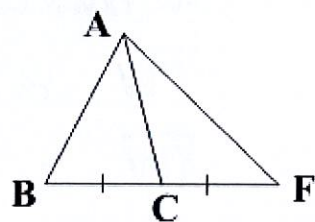
28.



$$\begin{aligned} 8x-63 &= 4x-27 \\ 4x &= 36 \\ \boxed{x=9} \end{aligned}$$

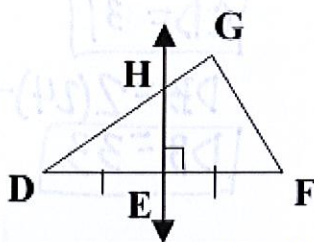
In #29-32, use one of our key terms from this chapter to name the identified segment.

29. $\overline{AC}$



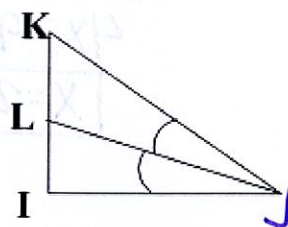
Median

30. $\overline{HE}$



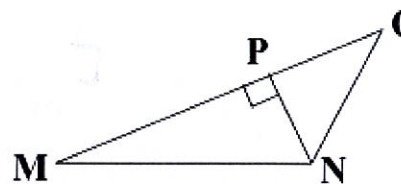
perpendicular bisector

31. $\overline{JL}$



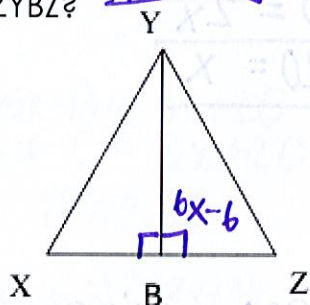
angle bisector

32. $\overline{PN}$



Altitude

33. $\overline{YB}$ is an altitude of $\triangle XYZ$ and $m\angle YBZ = (6x - 6)^\circ$. Find the value of x . What is the measure of $\angle YBZ$?



$$6x - 6 = 90$$

$$6x = 96$$

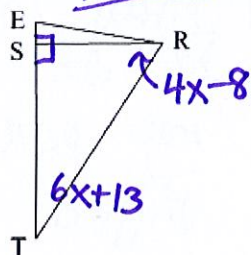
$$x = 16$$

$$m\angle YBZ = 6(16) - 6$$

$$m\angle YBZ = 90$$

↑
duh, it's an altitude!

34. $\overline{RS}$ is an altitude of $\triangle RTE$, $m\angle SRT = (4x - 8)^\circ$ and $m\angle STR = (6x + 13)^\circ$. Find the value of x .



$$4x - 8 + 6x + 13 + 90 = 180$$

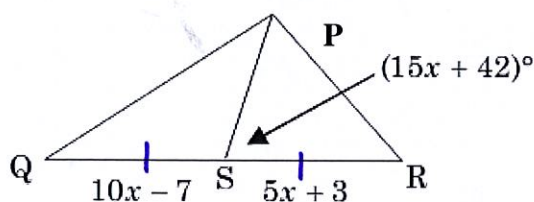
$$4x - 8 + 6x + 13 = 90$$

$$10x + 5 = 90$$

$$10x = 85$$

$$x = 8.5$$

35. Find x and the measure of $\angle PSR$ if $\overline{PS}$ is a median.



$$10x - 7 = 5x + 3$$

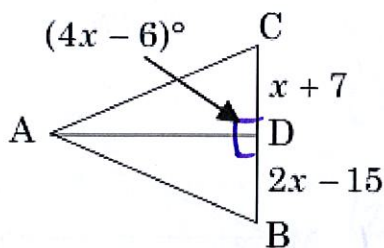
$$5x = 10$$

$$x = 2$$

$$m\angle PSR = 15(2) + 42$$

$$m\angle PSR = 72$$

36. Find x , CD , and DB if $\overline{AD}$ is an altitude of $\triangle ABC$.



$$4x - 6 = 90$$

$$4x = 96$$

$$x = 24$$

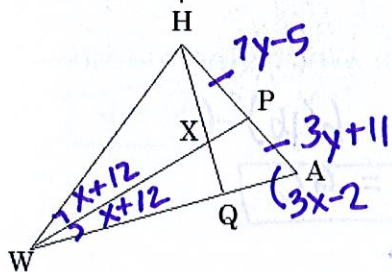
$$CD = 24 + 7$$

$$CD = 31$$

$$DB = 2(24) - 15$$

$$DB = 33$$

37. $\triangle WHA$, if $\overline{WP}$ is a median and an angle bisector, $AP = 3y + 11$, $PH = 7y - 5$, $m\angle HWP = x + 12$, $m\angle PAW = 3x - 2$, and $m\angle HWA = 4x - 16$, find x and y . Is $\overline{WP}$ also an altitude? Explain.



$$7y - 5 = 3y + 11$$

$$4y = 16$$

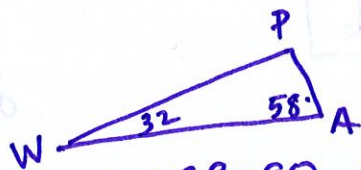
$$y = 4$$

$$x + 12 + x + 12 = 4x - 16$$

$$2x + 24 = 4x - 16$$

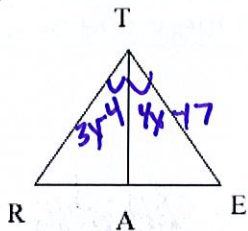
$$40 = 2x$$

$$20 = x$$



$32 + 58 = 90$ so $\angle P$ must be 90° making $\overline{WP}$ an altitude.

38. In $\triangle RTE$, $\overline{TA}$ bisects $\angle RTE$, $m\angle RTA = (3y - 4)^\circ$ and $m\angle ETA = (4y - 17)^\circ$. Find the measure of $\angle RTE$.



$$3y - 4 = 4y - 17$$

$$13 = y$$

$$m\angle RTE = 3(13) - 4 + 4(13) - 17$$

$$m\angle RTE = 39 + 35$$

$$m\angle RTE = 74$$