Honors Pre-Calculus **Midterm Exam Review** **Name:**
January 2014

Chapter 1: Functions and Their Graphs

1. Evaluate the function at each specified value of the independent variable and simplify.

$f(x) = x^2 + 1$

a. $f(-3)$ **b**. $f(x-1)$ **c**. $f(b^3)$

1a.)_____

1b.)_____

1c.)_____

2. Evaluate the function at each specified value of the independent variable and simplify.

$$f(x) = \begin{cases} 2x+1 & x \le -1 \\ x^2+2 & x > -1 \end{cases}$$

a. $f(-2)$ **b**. $f(-1)$ **c**. $f(0)$ **d**. $f(2)$

2a.)_____

2b.)_____

2c.)_____

2d.)_____

3. Find the domain of the given function. Use interval notation!

a. $f(x) = \dfrac{x-1}{x+2}$ **b**. $g(x) = \dfrac{x^2}{x^2+1}$ **c**. $f(x) = \sqrt{x^2-16}$

3a.)_____

3b.)_____

3c.)_____

4. Use a graphing utility to graph the function and determine the open intervals on which the function is increasing, decreasing, or constant (use interval notation!). Identify any relative minimum or relative maximum values of the function.

a. $f(x) = x^3 - 3x$ **b**. $f(x) = \sqrt{x^2-9}$

4a.) increasing:_____

decreasing:_____

constant:_____

rel. min.:_____

rel. max.:_____

4b.) increasing:_____

decreasing:_____

constant:_____

rel. min.:_____

rel. max.:_____

5. Determine algebraically if the function is even, odd, or neither. Verify using a graphing utility.

a. $f(x) = x^2 + 6$ **b**. $f(x) = x^2 - 6x + 4$ **c**. $f(x) = x^3 - 3x$

5a.)_____ **5b**.)_____ **5c**.)_____

6. Given $f(x)=x^2$, $g(x)=\sqrt{x}$ and $h(x)=|x|$. Describe the sequence of transformations from $f(x)$, $g(x)$, or $h(x)$ to $j(x)$.

a. $j(x)=-(x+2)^2-6$ **b**. $j(x)=2\sqrt{x}+3$ **c**. $j(x)=-\frac{1}{2}|x-5|-7$

a.)_____

b.)_____

c.)_____

7. Given $f(x)=3-2x$, $g(x)=\sqrt{x}$ and $h(x)=3x^2+2$. Find the indicated values.

a. $(f-g)(4)$ **b**. $(f+h)(5)$ **c**. $(fh)(1)$

d. $\left(\frac{g}{h}\right)(1)$ **e**. $(g\circ f)(-2)$ **f**. $(f\circ h)(-4)$

7a.)_____

7b.)_____

7c.)_____

7d.)_____

7e.)_____

7f.)_____

8. Find the inverse of the function algebraically.

a. $f(x)=\frac{7x+3}{8}$ **b**. $f(x)=4x^3-3$ **c**. $f(x)=\sqrt{x+10}$

8a.)_____

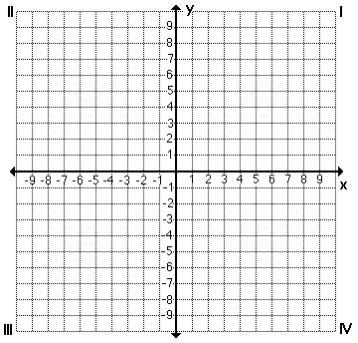
8b.)_____

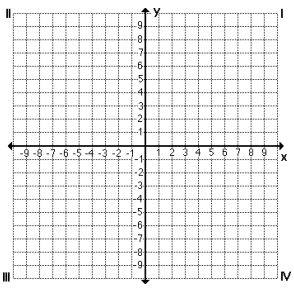
8c.)_____

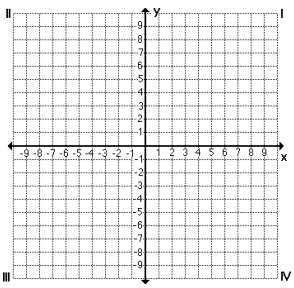
9. Show that $f(x)$ and $g(x)$ are inverses.

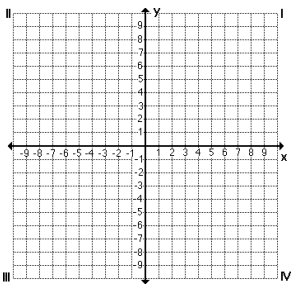
$f(x)=\sqrt{x+1}$, $g(x)=x^2-1$, $x\geq 0$

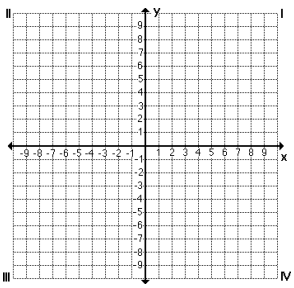
10. Given the graph $y=f(x)$ at the right, sketch the following:











a. $y=\frac{1}{2}f(x)$ **b**. $y=-f(x)$ **c**. $y=f(x-2)$ **d**. $y=|f(x)|$

11. Express each complex number in **standard form** $a+bi$.

a. $\sqrt{-12}-\sqrt{-48}$ **b**. $(3-2i)^2$ **c**. $\dfrac{1}{4-7i}$ **11a**.)_____

11b.)_____

11c.)_____

d. $\dfrac{\sqrt{3}+2i}{\sqrt{3}-2i}$ **e**. i^{23} **f**. $4(3+4i)-5i(1+i)$ **11d**.)_____

11e.)_____

11f.)_____

12. Solve the following equations for x.

a. $12x^2+12=25x$ **b**. $x^2+4x=7$ **c**. $x^2-6x+18=0$ **12a**.)_____

12b.)_____

12c.)_____

13. If $P(x)=4x^3-5x^2+1,$ use **synthetic substitution** to find:

a. $P(2)$ **b**. $P\left(\dfrac{1}{2}\right)$ **c**. $P(i)$ **13a**.)_____

13b.)_____

13c.)_____

14. If -2 is a zero of the polynomial $P(x)=2x^3+x+k,$ find the value of ***k***. **14**.)_____

15. A polynomial $P(x)$ is divided by $(x-2)$. The quotient is $2x^2+x-2$ and the remainder is -1. Find $P(x)$.

15.)_____

16. Two roots of the equation $x^4+x^3-5x^2+x-6=0$ are $x=2$ and $x=-3$. Find the remaining roots.

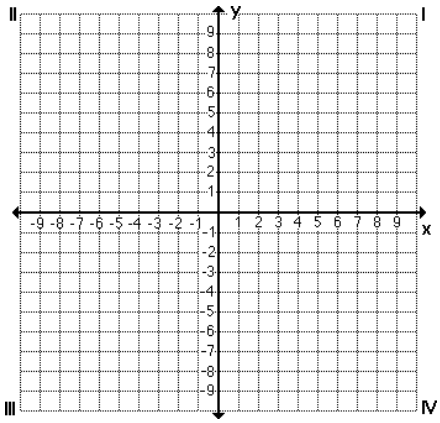
16.)_____

17. Find all real zeros of $f(x)=2x^4+7x^3-4x^2-27x-18$. Use the rational root theorem and synthetic division.

17.)_____

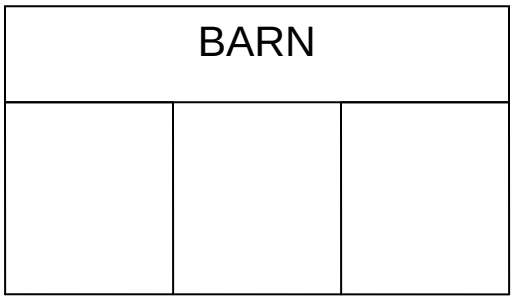
18. Sketch the graph of the equation (***without* a graphing calculator**). Use the **zeros** of the function and the **end behavior** of the function.

$f(x) = -x^2(x-1)(x+2)$



19. A rectangular enclosure, sub-divided into three congruent pens as shown, is to be made using a barn as one side and 120 meters of fencing for the rest of the enclosure. Find the value of x that gives the **maximum area** for the enclosure.

19.)_____

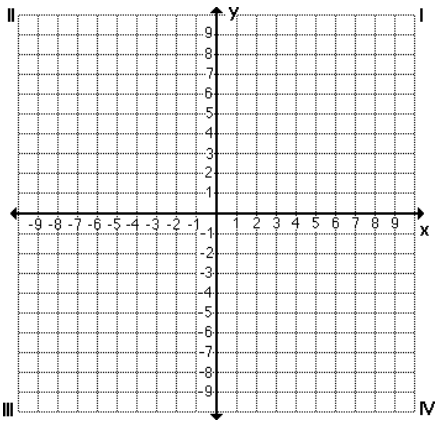


20. Solve for x. $2x^3 + 2x^2 - 4x - 4 = 0$ **20**.)_____

21. Find all **real** and **imaginary roots** of $2x^4 + 3x^3 + 7x^2 - 7x - 5 = 0$ **21**.)_____

22. Find a **cubic equation** with **integral coefficients** having the solutions $1+i, \text{and } 3$. **22**.)_____

23. Graph $f(x) = \begin{cases} x+2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$



24. Sketch the graph of the rational function by hand. Identify any **x-intercepts**, the **y-intercept**, **horizontal asymptotes**, **vertical asymptotes**, and **slant asymptotes**.

a. $f(x) = \dfrac{x-3}{x-2}$

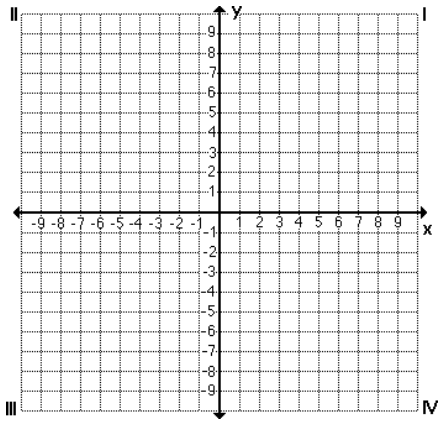
x-intercept(s) = _____

y-intercept = _____

H.A. = _____

V.A. = _____

S.A. = _____



b. $f(x) = \dfrac{2x^2}{x^2-4}$

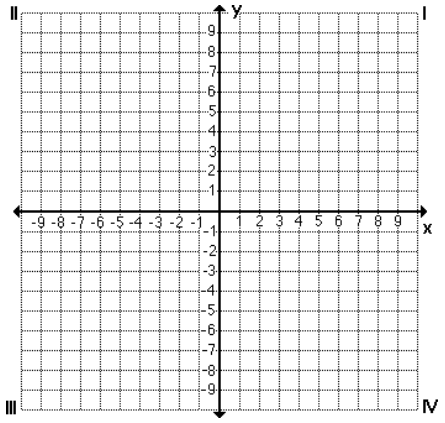
x-intercept(s) = _____

y-intercept = _____

H.A. = _____

V.A. = _____

S.A. = _____



c. $f(x) = \dfrac{x^2-x+1}{x-3}$

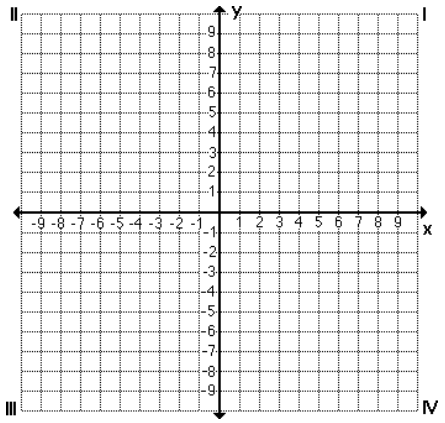
x-intercept(s) = _____

y-intercept = _____

H.A. = _____

V.A. = _____

S.A. = _____



25. Determine whether $x^2 - xy + y^2 = 6$ has **symmetry** in:
Circle the correct answer (s)

a. the x-axis **b**. the y-axis **c**. the line y=x **d**. the origin

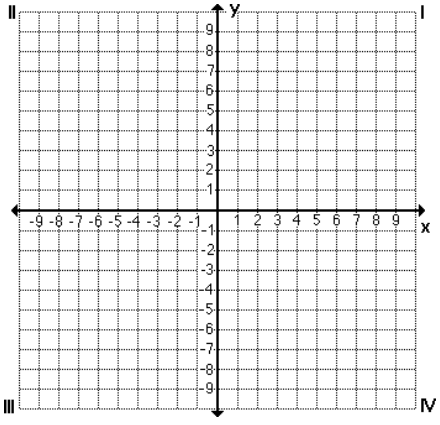
26. **Sketch** the graph of the quadratic function. Identify the **vertex** & **intercept(s)**.

$f(x) = (x-4)^2 - 4$

vertex = _____

x-intercept(s) = _____

y-intercept = _____



27. **Evaluate** the following exponential expressions:

a. $\dfrac{3^{-5}\cdot 3^{10}}{3^{2}}$ **b**. $\left(4^{-2}+4^{0}\right)^{-1}$ **c**. $\sqrt{\dfrac{9^{5}}{3^{-2}}}$ **d**. $\dfrac{2^{-4}+4^{-3}}{2^{-3}}$ **27a**.)_____

27b.)_____

27c.)_____

27d.)_____

28. **Solve** each exponential equation:

a. $3^{8-x}=27^{2-x}$ **b**. $4\sqrt{32}=2^{3x}$ **28a**.)_____

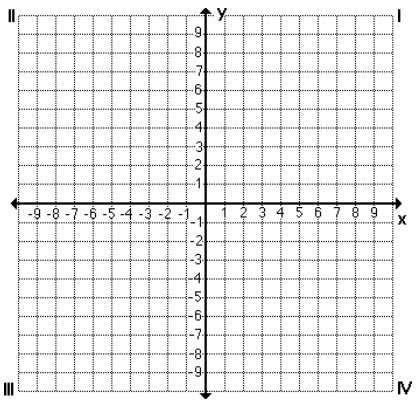
28b.)_____

29. A gallon of gasoline cost \$3.09 two years ago. Now it costs \$1.78. To the nearest percent, what has been the **annual rate of decrease** in cost?

29.)_____

30. **Graph** $y=3^{x}$ and $y=\left(\dfrac{1}{3}\right)^{x}$ on the same set of axes.

How are the graphs related?



31. Suppose \$1,200 is invested at an interest rate of 9.6%. how much is the investment worth after 2 years if interest is compounded:

a. **Monthly**? **b**. **Continuously**? **31a.**)_____

31b.)_____

32. Given $\log 40 \approx 1.6021$, **using rules of logarithms**, find the value of: {**without a calculator!}**

a. $\log 4$ b. $\log .25$ **32a**.)_____

32b.)_____

33. Find the **exact value** of: **without a calculator!**

33a.)_____

a. $\log_2 16$ b. $\log_{27} 3$ c. $5^{\log_4 64}$

33b.)_____

33c.)_____

34. Express the following in terms of $\log_b M$ and $\log_b N$

a. $\log_b \sqrt[3]{\frac{M^3}{N^2}}$ b. $\log_b M^3 N^2$ **34a**.)_____

34b.)_____

35. **Solve** $\log_2(x-2)+\log_2(x+5)=1.$ **35**.)_____

36. **Solve** $3^x = 31$ to the nearest hundredth. **36**.)_____

37. If 250 mg of ibuprofen has a half-life of 3 hours, then how much ibuprofen is in a person's bloodstream after (a) 7 hours? (b) after ***t*** hours?

37a.)_____

37b.)_____

38. **Simplify**:

a. $\frac{3^5 \bullet 3^{-3}}{3^{-2}}$ **38a**.)_____

b. $\sqrt[3]{\frac{8^{-1}}{8^{-7}}}$ **38b**.)_____

c. $\frac{6x^{\frac{-4}{3}} + 2x^{\frac{5}{3}}}{2x^{\frac{-1}{3}}}$ **38c**.)_____

39. **Solve** for x:

a. $\log_2 x = -4$ **39a**.)_____

b. $\log_3\left(x^2+5\right)=\log_3\left(4x^2-2x\right)$ **39b**.)_____

c. $\log_5 25\sqrt{5} = x$ **39c**.)_____

d. $\log_6 4 + \log_6 3 - \log_6 2 = x$ **39d**.)_____

40. **Solve** for x:

a. $e^{5x} = 2^{x-3}$ **40a**.)_____

b. $5^{x+1} = 2^{3x+6}$ **40b**.)_____

c. $e^{2x} - 3e^x + 2 = 0$ **40c**.)_____

d. $\sqrt{\frac{9^{x+3}}{27^x}} = 81$ **40d**.)_____

Chapter 4 [4.1-4.6]: Trigonometry

41. A circle has a radius of 7 inches. Find the **length of the arc** intercepted by a central angle of $240°$.

41.)_____

42. The circular blade on a saw rotates at 2400 revolutions per minute.

a. Find the **angular speed** in radians per second.

42a.)_____

b. The blade has a radius of 4 inches. Find the **linear speed** of a blade tip in inches per second.

42b.)_____

43. A satellite in circular orbit 1125 km above a planet makes one complete revolution every 120 minutes. Assuming that he planet is a sphere of radius 6400 km, find the linear speed of the satellite in **kilometers per minute**. Round your answer to the nearest whole number.

43.)_____

44. A truck is moving at a rate of 90 km per hour and the diameter of its wheels is 1.25 meters. Find the angular speed of the wheels in **radians per minute**.

44.)_____

45. **Evaluate** (if possible) the six trigonometric functions if $\theta = -\frac{2\pi}{3}$.

45. $\sin\left(-\frac{2\pi}{3}\right) =$ $\csc\left(-\frac{2\pi}{3}\right) =$

$\cos\left(-\frac{2\pi}{3}\right) =$ $\sec\left(-\frac{2\pi}{3}\right) =$

$\tan\left(-\frac{2\pi}{3}\right) =$ $\cot\left(-\frac{2\pi}{3}\right) =$

46. **Evaluate** the trigonometric function.

a. $\sin\left(-\frac{3\pi}{4}\right)$ **b**. $\csc\left(\frac{7\pi}{6}\right)$ **c**. $\tan\left(\frac{5\pi}{3}\right)$

46a.)_____

46b.)_____

46c.)_____

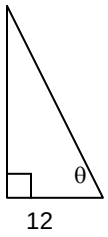
d. $\sec(-4\pi)$ **e**. $\cot\left(\frac{5\pi}{2}\right)$ **f**. $\cos\left(\frac{13\pi}{4}\right)$

46d.)_____

46e.)_____

46f.)_____

47. Find the **exact values** of the **six trigonometric functions** of the angle θ shown in the figure.



47. $\sin(\theta) =$ $\csc(\theta) =$

$\cos(\theta) =$ $\sec(\theta) =$

$\tan(\theta) =$ $\cot(\theta) =$

48. **Simplify** each expression.

a. $\dfrac{\sin x \cos x}{1-\cos^2 x}$ **b**. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$ **48a**.)_____

48b.)_____

c. $\dfrac{\tan^2 x}{\sec x + 1} + 1$ **d**. $\sec x - \sin x \tan x$ **48c**.)_____

48d.)_____

49. **Find two solutions** of the equation. Give your answers in degrees $(0° \le \theta < 360°)$ and radians $(0 \le \theta < 2\pi)$ without using a calculator.

a. $\cot\theta = -\dfrac{\sqrt{3}}{3}$ **b**. $\sec\theta = \sqrt{2}$ **49a**.)_____

49b.)_____

50. John stands 150 meters from a water tower and sights the top at an angle of elevation of $36°$. How tall is the tower?

50.)_____

51. The Aerial run in Snowbird, Utah has an angle of elevation of $20.2°$. Its vertical drop is 2900 feet. Estimate the length of the run.

51.)_____

52. In a sightseeing boat near the base of the Horseshoe Falls at Niagara Falls, a passenger estimates the angle of elevation to the top of the falls to be $30°$. If the Horseshoe Falls are 173 feet high, what is the distance from the boat to the base of the falls?

52.)_____

53. Use the given value and the trigonometric identities to **find the remaining trigonometric functions** of the angle.

$\cos\theta = -\frac{3}{7}, \quad \sin\theta < 0$

53. $\sin(\theta) =$ $\csc(\theta) =$

$\cos(\theta) =$ $\sec(\theta) =$

$\tan(\theta) =$ $\cot(\theta) =$

54. The point is on the terminal side of an angle in standard position. **Determine** the **exact values** of the six trigonometric functions of the angle.

$(8, -15)$

54. $\sin(\theta) =$ $\csc(\theta) =$

$\cos(\theta) =$ $\sec(\theta) =$

$\tan(\theta) =$ $\cot(\theta) =$

Answer the questions regarding the <u>amplitude</u>, <u>period</u>, <u>phase</u> <u>shift</u> and <u>vertical</u> <u>shift</u> of each trigonometric function $y = a\cos(b\theta - c) + d$ or $y = a\sin(b\theta - c) + d$ **. Then graph two full periods of the trigonometric function. Be sure to label your axes accurately!**

55. $y = \cos 4\theta - 2$

a = b = c = d =

amplitude = period = phase shift = vertical shift =

56. $y = -3\sin\left(\frac{\theta}{2}\right) + 1$

a = b = c = d =

amplitude = period = phase shift = vertical shift =

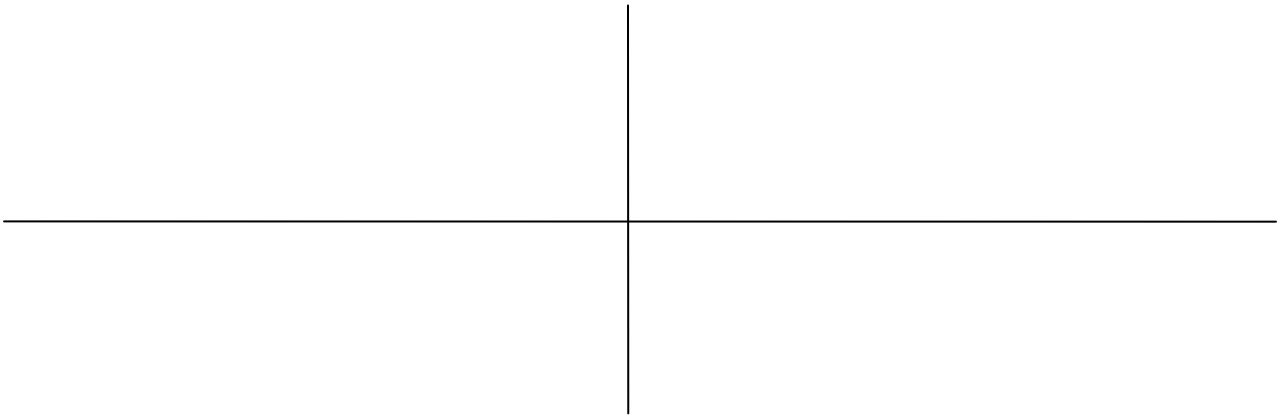
57. $y = 3\cos\left(\theta + \frac{\pi}{2}\right) + 4$

a = b = c = d =

amplitude = period = phase shift = vertical shift =

58. $y = 4\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 3$ a = b = c = d =

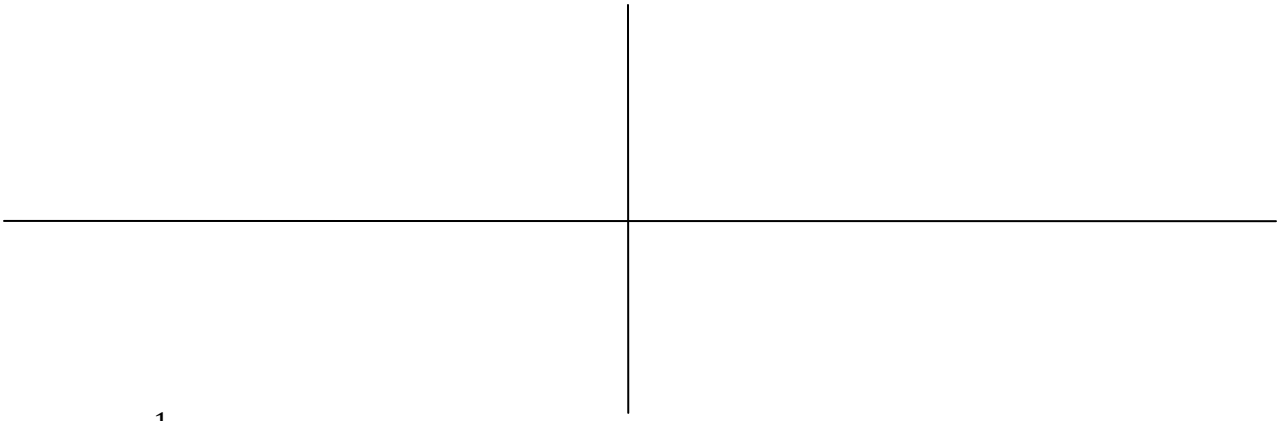
amplitude = period = phase shift = vertical shift =



Graph two periods of the trigonometric function.

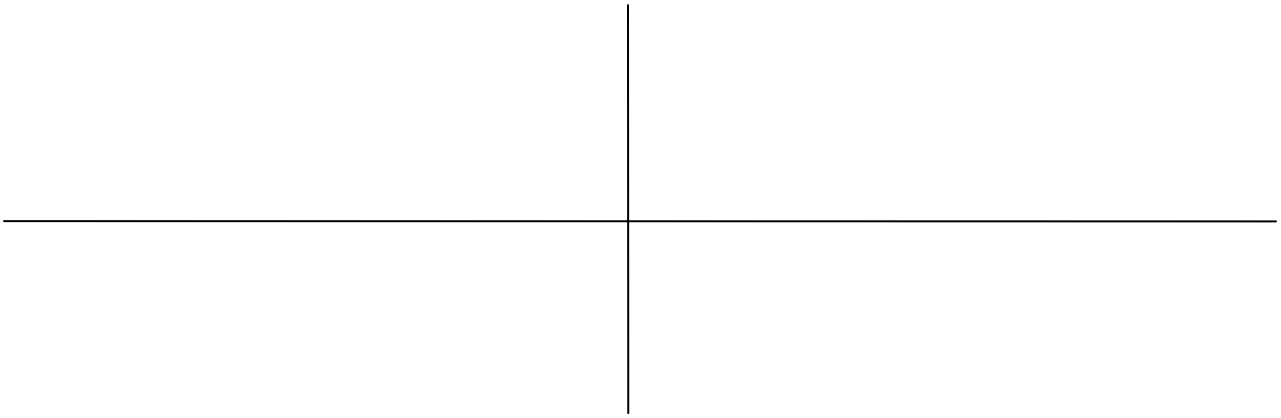
59. $y = -3\tan 2\theta$ a = b = c = d =

amplitude = period = phase shift = vertical shift =



60. $y = \frac{1}{2}\cot\theta + 1$ a = b = c = d =

amplitude = period = phase shift = vertical shift =



Answer Key Midterm Review Packet January 2013

1a. 10 **1b**. x^2-2x+2 **1c**. b^6+1 **2a**. -3

2b. -1 **2c**. 2 **2d**. 6 **3a**. $(-\infty, -2)\cup(-2, \infty)$

3b. $(-\infty, \infty)$ **3c**. $(-\infty, -4]\cup[4, \infty)$ **4a**. increasing: $(-\infty, -1)\cup(1, \infty)$

4a. decreasing: $(-1, 1)$ **4a**. constant: none **4a**. rel. min.: $(1, -2)$

4a. rel. max.: $(-1, 2)$ **4b**. increasing: $(3, \infty)$ **4b**. decreasing: $(-\infty, -3)$

4b. constant: none **4b**. rel. min.: $(-3, 0)$; $(3, 0)$ **4b**. rel. max.: none

5a. even **5b**. neither **5c**. odd

6a. $f(x)$ has a horizontal shift 2 units left, reflects over the x-axis and has a vertical shift 6 units down

6b. $f(x)$ has a vertical stretch of 2 and a vertical shift up 3 units

6c. $f(x)$ has a horiz. shift 5 units right, reflects over the x-axis, a vertical shrink of ½ and a vertical shift 7 units down

7a. -7 **7b**. 70 **7c**. 5 **7d**. $\frac{1}{5}$

7e. $\sqrt{7}$ **7f**. -97 **8a**. $f^{-1}(x)=\frac{8x-3}{7}$

8b. $f^{-1}(x)=\sqrt[3]{\frac{x+3}{4}}$ **8c**. $f^{-1}(x)=x^2-10,\ x\ge 0$ **9**. show that $f(g(x))=g(f(x))=x$

10. See graphs **11a**. $-2i\sqrt{3}$ **11b**. $5-12i$ **11c**. $\frac{4}{65}+\frac{7}{65}i$

11d. $-\frac{1}{7}+\frac{4\sqrt{3}}{7}i$ **11e**. $-i$ **11f**. $17+11i$ **12a**. $\frac{3}{4}, \frac{4}{3}$

12b. $-2\pm\sqrt{11}$ **12c**. $3\pm 3i$ **13a**. 13 **13b**. $\frac{1}{4}$

13c. $6-4i$ **14**. 18 **15**. $P(x)=2x^3-3x^2-4x+3$

16. $\pm i$ **17**. $-1, 2, -3, -\frac{3}{2}$ **18**. See graph **19**. 15 meters

20. $-1, \pm\sqrt{2}$ **21**. $1, -\frac{1}{2}, -1\pm 2i$ **22**. $f(x)=x^3-5x^2+8x-6$

23. See graph **24a**. x-intercept: $(3, 0)$; y-intercept: $\left(0, \frac{3}{2}\right)$; H.A.: y = 1; V.A.: x = 2; S.A.: none

24b. x-intercept: $(0, 0)$; y-intercept: $(0, 0)$; H.A.: y = 2; V.A.: x = 2 and x = -2; S.A.: none

24c. x-intercept: none; y-intercept: $\left(0, -\frac{1}{3}\right)$; H.A.: none; V.A.: x = 3; S.A.: x + 2

25. c and d **26**. vertex: $(4, -4)$; x-intercept: $(2, 0)$; $(6, 0)$; y-intercept: $(0, 12)$

27a. 27 **27b**. $\frac{16}{17}$ **27c**. 729 **27d**. $\frac{5}{8}$

28a. -1 **28b**. $\frac{3}{2}$ **29**. 24.1% **30**. See graph

31a. $1452.89 **31b**. $1454.00 **32a**. 0.6021 **32b**. -0.6021

33a. 4 **33b**. $\frac{1}{3}$ **33c**. 125 **34a**. $\log_b M-\frac{2}{3}\log_b N$

34b. $3\log_b M+2\log_b N$ **35**. $\frac{-3+\sqrt{57}}{2}$ **36**. 3.13 **37a**. ≈ 49.61

37b. $y=250\left(\frac{1}{2}\right)^{\frac{t}{3}}$ **38a**. 81 **38b**. 64 **38c**. $\frac{3+x^3}{x}$

39a. $\frac{1}{16}$ **39b**. $\frac{5}{3}, -1$ **39c**. $\frac{5}{2}$ **39d**. 1

40a. -0.483 **40b**. ≈ -5.412 **40c**. $0, 0.693(\ln 2)$ **40d**. -2

41. $\frac{28\pi}{3}$ inches **42a**. 80π rad/sec **42b**. ≈ 1005 in/sec **43**. 394 km/min

44. 2400 rad/min **45**. $\sin\left(-\frac{2\pi}{3}\right)=-\frac{\sqrt{3}}{2}$ $\csc\left(-\frac{2\pi}{3}\right)=-\frac{2\sqrt{3}}{3}$ **46a**. $-\frac{\sqrt{2}}{2}$

$\cos\left(-\frac{2\pi}{3}\right)=-\frac{1}{2}$ $\sec\left(-\frac{2\pi}{3}\right)=-2$

$\tan\left(-\frac{2\pi}{3}\right)=\sqrt{3}$ $\cot\left(-\frac{2\pi}{3}\right)=\frac{\sqrt{3}}{3}$

46b. -2 **46c**. $-\sqrt{3}$ **46d**. 1 **46e**. 0

46f. $-\frac{\sqrt{2}}{2}$ **47**. $\sin(\theta)=\frac{3\sqrt{13}}{13}$ $\csc(\theta)=\frac{\sqrt{13}}{3}$ $\cos(\theta)=\frac{2\sqrt{13}}{13}$ $\sec(\theta)=\frac{\sqrt{13}}{2}$ $\tan(\theta)=\frac{3}{2}$ $\cot(\theta)=\frac{2}{3}$ **48a**. $\cot x$ **48b**. 2

48c. $\sec x$ **48d**. $\cos x$ **49a.** $120°, 300°, \frac{2\pi}{3}, \frac{5\pi}{3}$

49b. $45°, 315°, \frac{\pi}{4}, \frac{7\pi}{4}$ **50**. 108.98 meters **51**. 8398.54 feet **52**. 299.64 feet

53. $\sin(\theta)=-\frac{2\sqrt{10}}{7}$ $\csc(\theta)=-\frac{7\sqrt{10}}{20}$ $\cos(\theta)=-\frac{3}{7}$ $\sec(\theta)=-\frac{7}{3}$ $\tan(\theta)=\frac{2\sqrt{10}}{3}$ $\cot(\theta)=\frac{3\sqrt{10}}{20}$ **54**. $\sin(\theta)=-\frac{15}{17}$ $\csc(\theta)=-\frac{17}{15}$ $\cos(\theta)=\frac{8}{17}$ $\sec(\theta)=\frac{17}{8}$ $\tan(\theta)=-\frac{15}{8}$ $\cot(\theta)=-\frac{8}{15}$ **55. – 60**. See graphs

55. a = 1 b = 4 c = 0 d = -2 Amp = 1 Per = $\frac{\pi}{2}$ Phase Shift = 0 Vert. Shift = -2

56. a = -3 b = $\frac{1}{2}$ c = 0 d = 1 Amp = 3 Per = 4π Phase Shift = 0 Vert. Shift = 1

57. a = 3 b =1 c = $-\frac{\pi}{2}$ d = 4 Amp = 3 Per = 2π Phase Shift = $-\frac{\pi}{2}$ Vert. Shift = 4

58. a = 4 b = $\frac{\pi}{2}$ c = -1 d = -3 Amp = 4 Per = 4 Phase Shift = -1 Vert. Shift = -3

59. a = -3 b = 2 c = 0 d = 0 Amp = none Per = $\frac{\pi}{2}$ Phase Shift = none Vert. Shift = none

60. a = $\frac{1}{2}$ b = 1 c = 0 d = 1 Amp = none Per = π Phase Shift = none Vert. Shift = 1