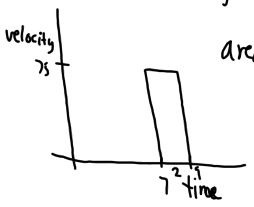
Integral Calculus

5.1 The Definite Integral

Ex) Train moves 75 mph from 7pm to 9pm. How far did it go?



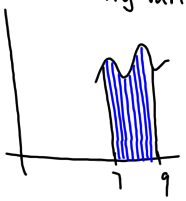
area = velocity x time

$= \frac{mi}{hr} \cdot hrs$

= miles

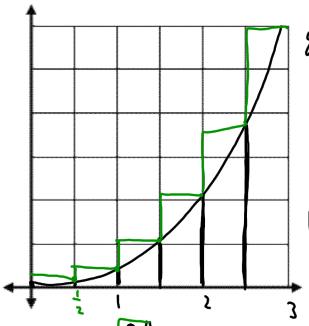
= 150 miles = distance

What if velocity varies?



Add up areas of all strips to approximate area

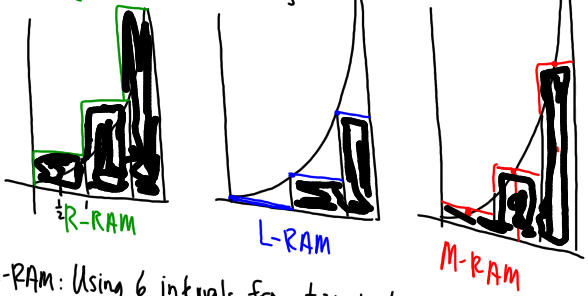
is almost



Ex) Particle starts at x=0 at t=0. Moves along x-axis with $v(t)=t^2$, $t \geq 0$.

Where is the particle when t=3?

Rectangular Approx Method (RAM)



R-RAM: Using 6 intervals from t=0 to t=3

$= \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2 + \frac{1}{2}\cdot(1)^2 + \frac{1}{2}\left(\frac{3}{2}\right)^2 + \frac{1}{2}(2)^2 + \frac{1}{2}\left(\frac{5}{2}\right)^2 + \frac{1}{2}(3)^2$

$= 11.375$

LRAM: $\frac{1}{2}\cdot(0)^2 + \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{2}(1)^2 + \frac{1}{2}\left(\frac{3}{2}\right)^2 + \frac{1}{2}(2)^2 + \frac{1}{2}\left(\frac{5}{2}\right)^2$

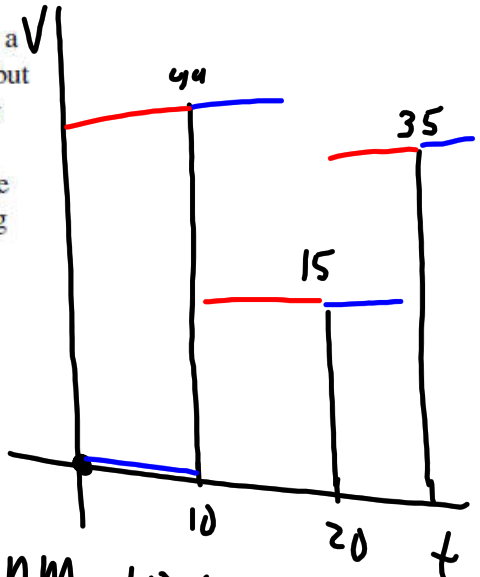
$= 6.875$

MRAM $\frac{1}{2}\cdot\left(\frac{1}{4}\right)^2 + \frac{1}{2}\left(\frac{3}{4}\right)^2 + \frac{1}{2}\left(\frac{5}{4}\right)^2 + \frac{1}{2}\left(\frac{7}{4}\right)^2 + \frac{1}{2}\left(\frac{9}{4}\right)^2 + \frac{1}{2}\left(\frac{11}{4}\right)^2$

$= 8.9375$

18. ***Length of a Road*** You and a companion are driving along a twisty stretch of dirt road in a car whose speedometer works but whose odometer (mileage counter) is broken. To find out how long this particular stretch of road is, you record the car's velocity at 10-sec intervals, with the results shown in the table below. (The velocity was converted from mi/h to ft/sec using 30 mi/h = 44 ft/sec.) Estimate the length of the road by averaging the LRAM and RRAM sums. 3665 ft

Time (sec)	Velocity (ft/sec)	Time (sec)	Velocity (ft/sec)
0	0	70	15
10	44	80	22
20	15	90	35
30	35	100	44
40	30	110	30
50	44	120	35
60	35		



LRAM: $10(0+44+\dots+30)$

RRAM: $10(44+15+\dots+35)$

$= 3665$ ft

Cardiac Output

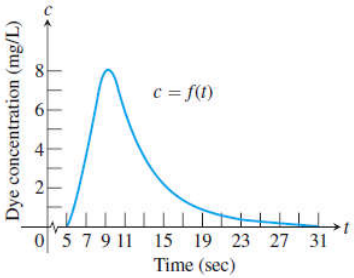
The number of liters of blood your heart pumps in a fixed time interval is called your *cardiac output*. For a person at rest, the rate might be 5 or 6 liters per minute. During strenuous exercise the rate might be as high as 30 liters per minute. It might also be altered significantly by disease. How can a physician measure a patient's cardiac output without interrupting the flow of blood?

One technique is to inject a dye into a main vein near the heart. The dye is drawn into the right side of the heart and pumped through the lungs and out the left side of the heart into the aorta, where its concentration can be measured every few seconds as the blood flows past. The data in Table 5.2 and the plot in Figure 5.10 (obtained from the data) show the response of a healthy, resting patient to an injection of 5.6 mg of dye.

Table 5.2 Dye Concentration Data

Seconds after Injection t	Dye Concentration (adjusted for recirculation) c
5	0
7	3.8
9	8.0
11	6.1
13	3.6
15	2.3
17	1.45
19	0.91
21	0.57
23	0.36
25	0.23
27	0.14
29	0.09
31	0

mg/L

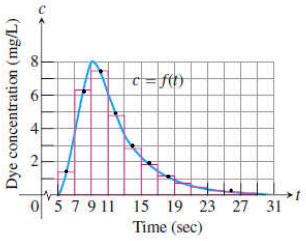


Know: 5.6 mg of dye

Want: L blood/min

area under curve: $\frac{\text{mg of dye}}{\text{L}} \cdot \text{sec}$

$$\frac{\text{mg of dye}}{\frac{\text{mg of dye}}{\text{L}} \cdot \text{sec}} = \frac{\text{mg of dye}}{1} \cdot \frac{\text{L}}{\text{mg of dye} \cdot \text{sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}}$$



$$\begin{aligned}\text{Area} &\approx f(6) \cdot 2 + f(8) \cdot 2 + f(10) \cdot 2 + \cdots + f(28) \cdot 2 \\ &\approx 2 \cdot (1.4 + 6.3 + 7.5 + 4.8 + 2.8 + 1.9 + 1.1 \\ &\qquad + 0.7 + 0.5 + 0.3 + 0.2 + 0.1) \\ &= 2 \cdot (27.6) = 55.2 \text{ (mg/L)} \cdot \text{sec}.\end{aligned}$$

$$\frac{5.6 \text{ mg}}{55.2 \text{ mg} \cdot \text{sec/L}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \approx 6.09 \text{ L/min}.$$

HW: p270 #5, 6, 15, 17, 19, 28

* calculator tomorrow