

**Learning Target:** 😊 Finding the zeros of a polynomial and their multiplicities, and using them along with end behavior, to sketch the graph of a polynomial by hand.

**Find the real zeros by factoring.**

**Example 1:**  $P(x) = x^3 - 6x^2 + 8x$

**Example 2:**  $P(x) = x^5 - 20x^3 + 64x$

**Intermediate Value Theorem:**

Use the **Intermediate Value Theorem** to verify that  $P(x)$  has a zero between  $a$  and  $b$ .

a.  $P(x) = 2x^3 - 21x^2 - 2x + 25$ ;  $a = 1$ ,  $b = 2$

b.  $P(x) = x^3 - x - 8$ ;  $a = 2.1$ ,  $b = 2.2$

❖ If when you plug in two values for  $x$  the signs change for  $y$ , what does that mean?

**Multiplicity of a Zero:**

**EVEN and ODD Powers of  $(x - c)$  Theorem:**

Multiplicity of the Zero is \_\_\_\_\_ the graph \_\_\_\_\_

Multiplicity of the Zero is \_\_\_\_\_ the graph \_\_\_\_\_

Determine the  $x$ -intercepts of the graph of  $P(x)$ . For each  $x$ -intercept, use the **EVEN and ODD Powers of  $(x - c)$  Theorem** to determine whether the graph of  $P(x)$  crosses the  $x$ -axis (**passes through**) or intersects but does not cross (**bounces off**) the  $x$ -axis. Also, determine the **multiplicity** of each  $x$ -intercept.

a.  $P(x) = -(x + 3)^2(x - 9)$

b.  $P(x) = (x + 2)(x - 6)^2$

\_\_\_\_\_ multiplicity \_\_\_\_\_ pass through/bounce

\_\_\_\_\_ multiplicity \_\_\_\_\_ pass through/bounce

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### Sketching polynomials WITHOUT TABLES OR GRAPHING CALCULATORS! ☺

#### Procedure for graphing:

1. Start by graphing the zeros
2. Then determine whether the graph passes through the zero or hits and bounces off the zero
3. Graph (if possible) the  $y$ -intercept
4. Determine the end behavior – which way should the arrows go?
5. Create a smooth curve connecting the intercepts

Sketch the graph of the polynomial function using end behavior, leading coefficients,  $x$ -intercepts and  $y$ -intercepts.

a.  $P(x) = x^3 + x^2 - 6x$   
[factor first!!]

What is the **degree** of the polynomial? \_\_\_\_\_

What **kind of number** is the degree? \_\_\_\_\_

What is the **sign** of the **leading coefficient**? \_\_\_\_\_

What is the **end behavior**?

As  $x \rightarrow \infty$ ,  $f(x) \rightarrow$

As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow$

What are the  $x$ -intercepts (zeros) & their multiplicities?

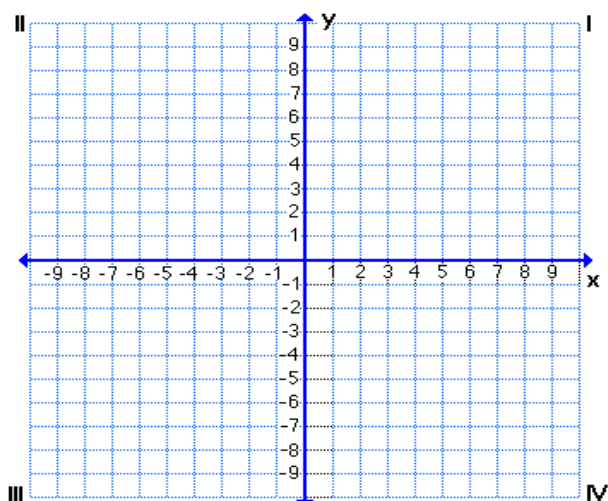
\_\_\_\_\_ multiplicity \_\_\_\_\_ pass through/bounce

\_\_\_\_\_ multiplicity \_\_\_\_\_ pass through/bounce

\_\_\_\_\_ multiplicity \_\_\_\_\_ pass through/bounce

What is the  $y$ -intercept? [plug in  $x = 0$ ] \_\_\_\_\_

***You will only graph this if it fits – if it doesn't, just estimate!***

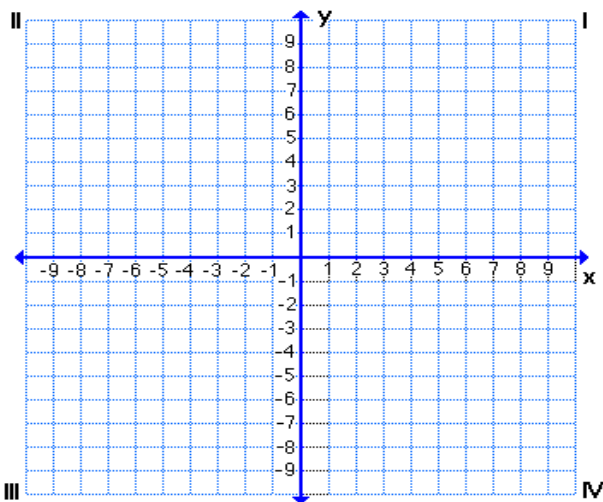


## Procedure for graphing:

1. Start by graphing the zeros
2. Then determine whether the graph passes through the zero or hits and bounces off the zero
3. Graph (if possible) the y-intercept
4. Determine the end behavior – which way should the arrows go?
5. Create a smooth curve

Sketch the graph of the polynomial function using end behavior, leading coefficients, x-intercepts and y-intercepts.

b.  $P(x) = -(x - 3)^2(x + 1)$



What is the **degree** of the polynomial? \_\_\_\_\_

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What is the **sign** of the **leading coefficient**? \_\_\_\_\_

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As  $x \rightarrow \infty$ ,  $f(x) \rightarrow$

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What are the x-intercepts (zeros) & their multiplicities?

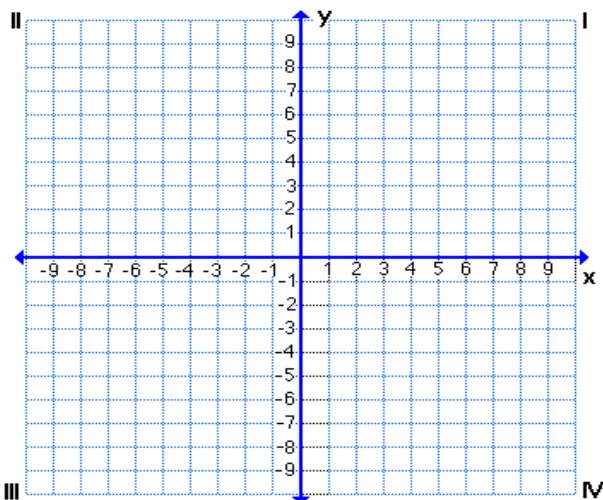
\_\_\_\_\_ multiplicity \_\_\_\_\_ pass through/bounce

\_\_\_\_\_ multiplicity \_\_\_\_\_ pass through/bounce

What is the y-intercept? [plug in  $x = 0$ ] \_\_\_\_\_

*You will only graph this if it fits – if it doesn't, just estimate!*

c.  $P(x) = -(x - 2)^2(x + 1)^2$



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What is the y-intercept? [plug in  $x = 0$ ] \_\_\_\_\_

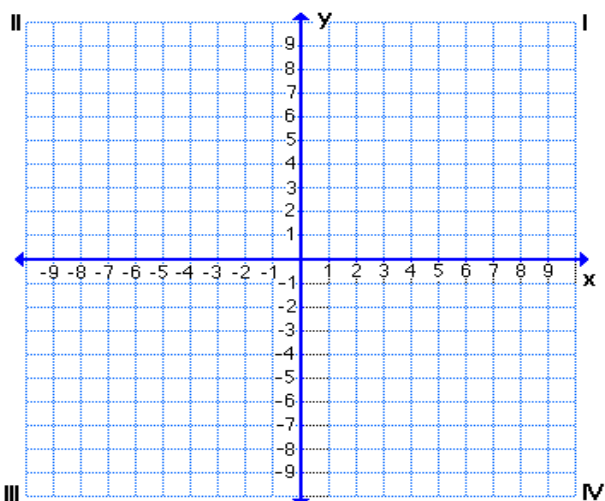
*You will only graph this if it fits – if it doesn't, just estimate!*



Key Ideas:

Do #41, 47 and 51 from page 284 right here!!

41.  $P(x) = x^3 - x^2 - 2x$



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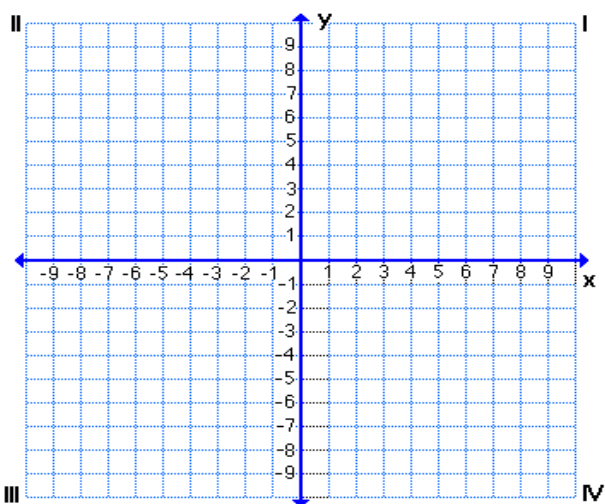
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What is the y-intercept? [plug in  $x = 0$ ] \_\_\_\_\_

47.  $P(x) = (x - 1)(x + 3)(x + 4)$



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What are the x-intercepts (zeros) & their multiplicities?

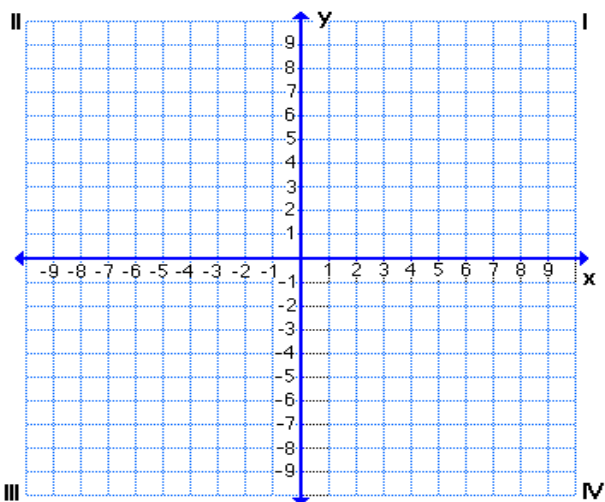
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\_\_\_\_\_ multiplicity \_\_\_\_\_ pass through/bounce

\_\_\_\_\_ multiplicity \_\_\_\_\_ pass through/bounce

What is the y-intercept? [plug in  $x = 0$ ] \_\_\_\_\_

51.  $P(x) = -x(x - 2)^2$



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What are the x-intercepts (zeros) & their multiplicities?

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\_\_\_\_\_ multiplicity \_\_\_\_\_ pass through/bounce

What is the y-intercept? [plug in  $x = 0$ ] \_\_\_\_\_