1. If $\sin \theta = -\frac{2}{3}$, and $\cos \theta < 0$ find the other five trig functions. No Calculator.

$$\sin \theta = -\frac{2}{3}$$
$$\cos \theta = \frac{\sqrt{5}}{3}$$
$$\tan \theta = -\frac{2\sqrt{5}}{5}$$
$$\csc \theta = -\frac{3}{2}$$
$$\sec \theta = -\frac{3\sqrt{5}}{5}$$
$$\cot \theta = \frac{\sqrt{5}}{2}$$

Find the exact value for $x$ and $y$. No calculator.

2. $x = \frac{5\sqrt{3}}{3}$
$y = \frac{28\sqrt{3}}{3}$

3.

$$\frac{5\sqrt{3}}{3} + \frac{5\sqrt{3}}{3}$$
$$\frac{20\sqrt{3}}{3}$$

4.

$$x = \sqrt{8}$$

5. Find the perimeter of the triangle below.

$$18\sqrt{2}$$

Draw a picture for each! Round all answers to the nearest thousandth. You MAY NOT use Law of Sines or Law of Cosines for this section.

6. A woman standing on a hill sees a flagpole that she knows is 60 feet tall. The angle of depression from her line of sight to the bottom of the pole is $14^\circ$, and the angle of elevation from her line of sight to the top of the pole is $18^\circ$. Find her distance from the pole.

$$\tan 18 = \frac{x}{y}$$
$$\tan 14 = \frac{60-x}{y}$$

$$y \tan 18 = x$$
$$y \tan 14 = 60 - x \Rightarrow x = 60 - y \tan 14$$

$$y = 104.485 \text{ feet}$$
7. To measure the height of the cloud cover at an airport, a worker shines a spotlight upward at an angle of 75° from the horizontal. An observer 600 m away on the other side of the cloud, measures the angle of elevation to the spot of light to be 45°. Find the height of the cloud cover.

\[ \tan 75° = \frac{x}{600-x} \]

\[(600-x) \tan 75° = x \]

\[2239.23 - 3.73x = x \]

\[2239.23 = 4.73x \]

\[x = 473.205 \text{ m} \]

8. A trapezoid has bases 4 cm and 12 cm with base angles 30° and 60°. Find the perimeter (exactly).

\[ x\sqrt{3} + \frac{x\sqrt{3}}{3} + 4 = 12 \]

\[ \frac{3x\sqrt{3}}{3} + \frac{x\sqrt{3}}{3} = 8 \]

\[ x\sqrt{3} = 4 \]

\[ x = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3} = 2\sqrt{3} \]

Perimeter = \(20 + 4\sqrt{3}\)

Find the sign of the expression if the terminal point determined by \(t\) is in the given quadrant.

9. \(\cot \theta \csc^2 \theta;\) QIII

\[ (+(-))^2 \]

\[ + + = + \]

10. \(\frac{\tan x}{\sin x \sec x} ;\) QII

\[ (+)(-)(-) \]

\[ = (-) \]

From the information given, find the quadrant in which \(\theta\) lies.

11. \(\tan x > 0 \text{ and } \sin x < 0\)

12. \(\csc \theta > 0 \text{ and } \sec \theta < 0\)

Find the exact value of the function. No Calculator.

13. \(\tan \frac{5\pi}{2}\)

\[ \frac{\sin}{\cos} = \frac{4}{x} = \frac{1}{0} \]

14. \(\sin \frac{2\pi}{3}\)

15. \(\sin \frac{3\pi}{4}\)

16. \(\csc \frac{7\pi}{4}\)

17. \(\tan 315°\)

18. \(\cos \frac{10\pi}{3}\)

Solve the following triangles using the Law of Sines or Cosines. Round answers to the nearest tenth.

19. \(A = 110°, \ a = 18, \ b = 12\)

20. \(A = 36°, \ a = 6, \ b = 8\)
21. \( B = 150^\circ, a = 100, c = 200 \)

\[
\begin{align*}
b^2 &= 100^2 + 200^2 - 2(100)(200)\cos 150^\circ \\
&= 250000 - 40000 \cos 150^\circ \\
b^2 &= 84441.016; b = 290.93
\end{align*}
\]

\[
\begin{align*}
\sin 150^\circ &= \frac{\sin C}{290.93} \\
A &= 9.9^\circ \\
C &= 20.1^\circ
\end{align*}
\]

\[
\begin{align*}
\frac{\sin 150^\circ}{290.93} &= \frac{\sin C}{200} \\
C &= 20.1^\circ
\end{align*}
\]

**Find the areas of the following triangles. Round answers to the nearest tenth.**

23. \( A = 25^\circ, B = 50^\circ, b = 30 \)

24. \( C = 45^\circ, a = 18, b = 10 \)

25. \( a = 7, b = 12, c = 8 \)

\[
\begin{align*}
K &= \frac{1}{2}(30)^2 \frac{\sin 15^\circ}{\sin 50^\circ} \\
&= 239.8
\end{align*}
\]

\[
\begin{align*}
K &= \frac{1}{2}(18)(10)\sin 45^\circ \\
&= 63.6
\end{align*}
\]

\[
\begin{align*}
K &= \sqrt{13.5(13.5 - 7)(13.5 - 12)(13.5 - 5)} \\
&= 26.9
\end{align*}
\]

**Solve the following. In the previous section, you HAD to use Right Triangles. In this section, you may use Right Triangles or the Laws of Sines and Cosines.**

26. Two snowmobilers start from the same point and drive at 10 km/h and 12 km/h, respectively, diverging at an angle of 110°. Two hours after leaving, they find that their radio transmissions are barely audible. How far apart are they at that time? **Round to the nearest thousandth.**

\[
\begin{align*}
X^2 &= 20^2 + 24^2 - 2(20)(24)\cos 110^\circ \\
&= 512 \cos 110^\circ \\
&= 36.116 \text{ km}
\end{align*}
\]

27. Jan is flying a plane on a triangular course at 320 mi/h. She flies due east for two hours and then turns right through a 65° angle. How long after turning will she be exactly southeast of where she started?

\[
\begin{align*}
\frac{\sin 20^\circ}{640} &= \frac{\sin 45^\circ}{X} \\
X &= \frac{1323.162828}{320} = 4.13488 = 4 \text{ hours and 8 minutes}
\end{align*}
\]

28. In quadrilateral ABCD, \( AB = 3, BC = 4, CD = 5, \) and \( DA = 6. \) The length of diagonal BD is 7. Find the length of the other diagonal. **Round to the nearest tenth.**

\[
\begin{align*}
&\quad 6^2 = 3^2 + 7^2 - 2(3)(7)\cos \chi \\
&\quad 36 = 58 - 42 \cos \chi \\
&\quad -22 = -42 \cos \chi; \quad \chi = 58.41^\circ
\end{align*}
\]

\[
\begin{align*}
&\quad 5^2 = 4^2 + 7^2 - 2(4)(7)\cos \gamma \\
&\quad 25 = 65 - 56 \cos \gamma \\
&\quad -40 = -56 \cos \gamma; \quad \gamma = 44.42^\circ
\end{align*}
\]

\[
\begin{align*}
AC &= \sqrt{3^2 + 4^2 - 2(3)(4)\cos 58.41^\circ} \\
&= 2.5 - 24 \cos 102.83^\circ \\
&= 30.329417 \\
AC &= 3.51
\end{align*}
\]
29. Find CD: Round to the nearest hundredth.

\[
\begin{align*}
\frac{\sin 116}{100} &= \frac{\sin 40}{x} & \frac{\sin 116}{100} &= \frac{\sin 24}{y} \\
x &= 71.52 & y &= 45.25 \\
\frac{\sin 88.5}{71.52} &= \frac{\sin 31}{z} & \frac{\sin 71}{45.25} &= \frac{\sin 45}{k} \\
z &= 36.98 & k &= 33.84 \\
DC^2 &= (36.98)^2 + (33.84)^2 - 2(36.98)(33.84)\cos 116 \\
DC^2 &= 2512.666 - 2562.8064 \cos 116 \\
DC^2 &= 3609.824 \\
DC &= 60.08
\end{align*}
\]

30. Two balloons are moored directly over a straight, level road. The diagram shows the angle of elevation of the balloons from two observers on the road one kilometer apart. How far apart are the balloons? Which balloon is higher, and by how many meters?

\[
\begin{align*}
\frac{\sin 13\theta}{1} &= \frac{\sin 25}{y} & y &= 1.608 \\
\frac{\sin 13\theta}{1} &= \frac{\sin 19}{z} & z &= 0.469 \\
\frac{\sin 40}{1.608} &= \frac{\sin 13\theta}{k} & k &= 0.941 \\
\frac{\sin 23}{0.469} &= \frac{\sin 113}{m} & m &= 1.105 \\
X^2 &= (0.941)^2 + (1.105)^2 - 2(0.941)(1.105)\cos 13\theta \\
X^2 &= 3.602 \\
X &= 1.898 \text{ km}
\end{align*}
\]

31. Find the area of a parallelogram that has a 72° angle and sides with lengths 7 and 15.

\[
K = \frac{1}{2} (7)(15) \sin 72
K = 49.930467 (x2) 
\]

32. Find the area of a rhombus that has a perimeter 32 and a 40° angle.

\[
K = 2\left(\frac{1}{2}\right)(8)(8) \sin 40
K = 41.138
\]

33. An arc of length 100 m subtends a central angle \(\theta\) in a circle of radius 50 m. Find the measure of \(\theta\) in radians and degrees.

\[
100 = 50\theta \\
\theta = 2 \text{ radians} \\
\theta = 114.6^\circ
\]
34. Find the area of a sector with central angle $80^\circ$ in a circle of radius 8 mi.

\[
80^\circ = \frac{4\pi}{9}
\]

\[
A = \frac{1}{2} (8)^2 \left( \frac{4\pi}{9} \right) = 44.68 \text { mi}^2
\]

35. The area of a circle is 72 cm$^2$. Find the area of a sector of this circle that subtends a central angle of $\frac{\pi}{6}$ rad.

\[
A = \frac{\pi r^2}{2} = \frac{72}{2} \quad \Rightarrow \quad r = 4.787
\]

\[
A = \frac{1}{2} (4.787)^2 \left( \frac{\pi}{6} \right) = 6 \text { cm}^2
\]

36. The wheels of a car have a diameter of 22 in and are rotating at 600 rpm. Find the speed of the car in mi/hr.

\[
\frac{r \Theta}{t} = \frac{11 \text { inches} \times 1200 \pi}{1 \text { min}} \times \frac{1 \text { mi}}{63360 \text { inches}} \times \frac{60 \text { min}}{1 \text { hr}} = 89.27 \text { mi/hr}
\]

37. The earth rotates about its axis once every 23 h 56 min 4 s, and the radius of the earth is 3960 mi. Find the linear speed of a point on the equator in mi/hr.

\[
\frac{r \Theta}{t} = \frac{3960 \text { mi} \times 2\pi}{23.934 \text { hrs}} = 1039.37 \text { mi/hr}
\]

38. The carousel at the county fair makes 3 revolutions per minute.

a) Find the linear speed in ft/sec of a person riding a horse that is 22.5 ft from the center.

\[
\frac{r \Theta}{t} = \frac{22.5 \times 60 \pi}{1 \text { min}} \times \frac{1 \text { min}}{60 \text { sec}} = 7.07 \text { ft/sec}
\]

b) The linear speed of the person on the inside of the carousel is 3.1 ft/sec. How far is this person from the center?

\[
3.1 = \frac{r \times 60 \pi}{60 \text { sec}} \quad \Rightarrow \quad r = 9.87 \text { ft}
\]

c) How much faster is the rider on the outside going than the rider on the inside?

\[
7.1 - 3.1 = 4 \quad \text{ft/sec faster}
\]