Write a formula for the measures of all angles coterminal with the given angle. Then use the formula to find two angles, one positive and one negative, that are coterminal with the given angle.

1. \( \theta = 670^\circ \)
   \[ \theta = 670^\circ - 360^\circ n \quad \text{where } n \in \mathbb{Z} \]
   \[ 310^\circ, -50^\circ \]

2. \( \frac{13\pi}{8} \)
   \[ \theta = \frac{13\pi}{8} + 2\pi n \quad \text{where } n \in \mathbb{Z} \]
   \[ \frac{29\pi}{8}, -\frac{3\pi}{8} \]

Express in either “Decimal Degrees” or in “Degrees Minutes Seconds” to the nearest second.

3. 23°27'8"
   \[ 23.45^\circ \]

4. 102°39'24"

Find a first-quadrant angle \( \theta \), for which an angle five times as large as \( \theta \) will be in the given quadrant.

5. Quadrant 2
   \[ 90^\circ < \theta < 180^\circ \]
   \[ 18^\circ < \theta < 36^\circ \]

6. Quadrant 4
   \[ 270^\circ < \theta < 360^\circ \]
   \[ 54^\circ < \theta < 72^\circ \]

Find the six trig functions of \( \theta \)

7. \( \theta = \frac{2}{5} (\sqrt{15})^2 + x^2 \)
   \[ x^2 = 19 \]

8. If \( \cos \theta = \frac{2}{7} \) and \( \csc \theta < 0 \) find the other five trig functions.

   \( \sin \theta = -\frac{3\sqrt{15}}{7} \)
   \( \csc \theta = -\frac{7}{3\sqrt{15}} \)
   \( \cos \theta = \frac{2}{7} \)
   \( \sec \theta = \frac{7}{2} \)
   \( \tan \theta = -\frac{3\sqrt{15}}{2} \)
   \( \cot \theta = -\frac{2\sqrt{15}}{3} \)

Find the exact value for \( x \) and \( y \).

9. \( x = 2 \sqrt{3} \)

10. \( x = \frac{13}{2} \)

11. \( y = \sqrt{3} \)

12. \( x = 24 \)

13. \( x = 4\sqrt{6} \)

   \( y = 3\sqrt{3} \)

   \( x = \frac{76\sqrt{3}}{3} \)

   \( y = \sqrt{x} \)

   \( y = 4\sqrt{3} \)
Draw a picture for each! Round all answers to the nearest thousandth.

14. An airplane is at an elevation of 45,000 ft when it begins its approach to an airport. Its angle of descent is 3°. What is the approximate air distance between the plane and the airport?

\[
\sin 3° = \frac{45000}{x}
\]

\[x = 8598.29517 \text{ ft}\]

15. A window washer 50 ft above the ground sees a parked car 153 ft away. What is the angle of depression from the man to the car?

\[
\cos y = \frac{50}{153}
\]

\[y = 70.925°\]

\[x = 19.075°\]

16. Find the measures of the angles of an isosceles triangle whose sides are 5, 10, and 10.

\[
\cos x = \frac{2.5}{10}
\]

\[x = 75.522°, 75.522°, 28.955°\]

17. Two farmers stand on the same side of a silo 10 feet apart. The angles of elevation to the top of the silo are 25° and 30° respectively. How far is each farmer from the silo? How high is the silo?

\[
y \tan 30° = \frac{x}{y} + 10\tan 25° = x
\]

\[y \tan 30° = y \tan 25° + 10 \tan 25° = x
\]

\[y \tan 30° = 0.577y = 0.466y + 4.1603
\]

\[y = 41.99357
\]

18. While traveling across flat land, you notice a mountain directly in front of you. The angle of elevation to the peak is 7°. After you drive 10 miles closer to the mountain, the angle of elevation is 18°. Approximate the height of the mountain.

\[
y \tan 18° = \frac{x}{y} + 10\tan 7° = x
\]

\[y \tan 18° = y \tan 7° + 10 \tan 7° = x
\]

\[y = 6.074 \text{ mi}
\]

\[x = 1.974 \text{ mi}
\]

Convert from Radians to Degrees, or Degrees to Radians

19. 30° = \frac{\pi}{180}

20. \frac{11\pi}{3} = \frac{180}{\pi}

21. 225° = \frac{\pi}{150}

22. \frac{2\pi}{4} = \frac{150}{11
From the information given, find the quadrant in which $\theta$ lies.

23. $\sin \theta > 0$ and $\tan \theta < 0$

24. $\csc \theta > 0$ and $\sec \theta < 0$

Find the exact value of the function without a calculator.

25. $\sec 150^\circ$

26. $\tan \frac{3\pi}{4}$

27. $\sin 300^\circ$

28. $\csc \frac{3\pi}{2}$

29. $\cot 30^\circ$

30. $\cos \frac{11\pi}{6}$

31. Find the value of all six trig functions at each quadrant angle.

$90^\circ$: $\sin 1$, $\cos 0$, $\tan$ undef, $\sec 1$, $\csc$ undef.
$180^\circ$: $\sin 0$, $\cos -1$, $\tan$ undef, $\sec -1$, $\csc$ undef.
$270^\circ$: $\sin -1$, $\cos 0$, $\tan$ undef, $\sec$ undef, $\csc 1$, $\csc$ undef.
$360^\circ$: $\sin 0$, $\cos 1$, $\tan$ undef, $\sec 1$, $\csc$ undef.

Express as the function of an acute angle.

32. $\cos 330^\circ$

33. $\sin 225^\circ$

34. Find the $\cot \theta$ if $\sin \theta = \frac{3}{5}$ and $\cos \theta < 0$.

35. Find $\cos x = -\frac{\sqrt{2}}{2}$ when $-\pi < x < 0$.

Are the following points on the unit circle? Show your work.

36. $\left( \frac{21}{29}, -\frac{20}{29} \right)$

$x^2 + y^2 = 1$

$\left( \frac{21}{29} \right)^2 + \left( -\frac{20}{29} \right)^2 = 1$

$\checkmark$ yes

37. $\left( -\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10} \right)$

$x^2 + y^2 = 1$

$\left( -\frac{\sqrt{10}}{10} \right)^2 + \left( \frac{3\sqrt{10}}{10} \right)^2 = 1$

$\frac{10}{100} + \frac{90}{100} = 1$  $\checkmark$ yes
Solve the following triangles.

38. \( A = 29^\circ, a = 6, b = 13 \)

\[ \frac{\sin 29}{6} = \frac{\sin 6}{13} \]

No Solution

39. \( A = 66^\circ, a = 12, b = 7 \)

\[ \frac{\sin 66}{12} = \frac{\sin 81.8}{13.6} \]

\[ B = 32.2^\circ, C = 81.8^\circ, C = 13.0 \]

40. \( A = 45^\circ, a = 15, b = 18 \)

\[ \frac{\sin 45}{15} = \frac{\sin 76.9}{18} \]

\[ B = 58.1^\circ, C = 76.9^\circ, C = 20.7 \]

41. \( a = 8, b = 6, c = 9 \)

\[ 9^2 = 6^2 + 8^2 - 2(\times 8)\cos C \]

\[ -19 = -96\cos C \]

\[ C = 78.6^\circ \]

\[ \frac{\sin 78.6}{c} = \frac{\sin B}{c} \]

\[ B = 40.8^\circ \]

42. In \( \triangle RST, R = 49.8^\circ, S = 67.5^\circ, r = 16 \)

Find \( t \)

\[ \frac{\sin 49.8}{16} = \frac{\sin 62.7}{t} \]

\[ t = 18.6 \]

43. In \( \triangle RST, T = 75^\circ, s = 20, r = 15 \)

Find \( t \)

\[ t^2 = 15^2 + 20^2 - 2(15 \times 20)\cos 75 \]

\[ t^2 = 625 - 600\cos 75 \]

\[ t^2 = 469.7086 \]

\[ t = 21.7 \]

44. Two snowmobilers start from the same point and drive at 8 km/h and 13 km/h, respectively, diverging at an angle of 100°. Three hours after leaving, they find that their radio transmissions are barely audible. How far apart are they at that time? Round to the nearest thousandth.

\[ X^2 = 24^2 + 39^2 - 2(24)(39)\cos 100 \]

\[ X^2 = 2097 - 1872\cos 100 \]

\[ X = 419.215 \text{ km} \]

45. A triangle has sides of lengths 7, 12 and 10. Find the measure of the smallest angle to the nearest tenth of a degree.

\[ 12^2 = 7^2 + 10^2 - 2(7)(10)\cos A \]

\[ -19 = -140\cos A \]

\[ A = 87.953° \]

\[ \frac{\sin 87.953}{12} = \frac{\sin C}{7} \]

\[ C = 35.659° \]
46. Jan is flying a plane on a triangular course at 450 mi/h. She flies due east for four hours and then turns right through a 50° angle. How long after turning will she be exactly southeast of where she started?

\[
\begin{align*}
1800 \text{ mi} & \quad \sin 50° = \sin 45° \\
\frac{1800}{x} & = \frac{\sin 45°}{x} \\
x & = 1460.365279 \text{ mi}
\end{align*}
\]

32 hrs 27 minutes

47. Find the length of an arc that subtends a central angle of 45° in a circle with radius 10 m.

\[
S = r \theta = 10(\frac{\pi}{4}) = 7.85 \text{ m}
\]

48. Find the area of a sector with central angle 60° in a circle with radius 3 mi.

\[
A = \frac{1}{2} (3^2) \frac{\pi}{3} = 4.71 \text{ mi}^2
\]

49. A woman is riding a bicycle whose wheels are 28 in. in diameter. If the wheels rotate at 130 revolutions per minute (rpm), find the speed at which she is traveling in mi/h.

\[
V = \frac{\pi r}{12} \frac{14(260 \pi)}{1 \text{ min}} = \frac{1 \text{ mi}}{5280 \text{ ft}} \frac{60 \text{ min}}{1 \text{ hr}} = 10.83 \text{ mi/hr}
\]

50. A boy rotates a stone in a 3 ft. long sling at the rate of 15 revolutions every 10 seconds. Find the linear and angular velocities of the stone.

\[
\omega = \frac{30 \pi}{10 \text{ sec}} = 9.42 \text{ radians/sec}
\]

\[
V = \frac{30 \pi \cdot 3 \text{ ft}}{10 \text{ sec}} = 28.27 \text{ ft/sec}
\]

Determine whether each function is even, odd, or neither.

51. \( f(x) = \frac{1}{6} + x \)


52. \( f(x) = x^2 + 3x \)

\( (-x)^2 + 3(-x) \)

\( x^2 - 3x \)

53. \( f(x) = x^8 \)

\( (-x)^8 \)

\( x^8 \)

54. \( f(x) = x^7 \)

\( (-x)^7 \)

\( -x^7 \)

Neither

Neither

Even

Odd

55. \( y = 3 \cos 3x \)

Amplitude: 3

Period: \( \frac{2\pi}{3} \)

Phase Shift:

Vertical Shift:
56. \( y = -\frac{1}{2} \sin \frac{x}{2} \)

- Amplitude: \( \frac{1}{2} \) \( \omega \) \( \text{ref.} \)
- Period: \( 2\pi \times \frac{1}{2} = \pi \)
- Phase Shift:
- Vertical Shift:

\[
\sin^2\left(x - \frac{\pi}{2}\right) + 1
\]

57. \( y = \sin(2x - \pi) + 1 \)

- Amplitude: 1
- Period: \( \frac{2\pi}{2} = \pi \)
- Phase Shift: \( \pi \times \frac{1}{2} \)
- Vertical Shift: Up 1

\[
2 \cos 2 \pi (x - \frac{1}{2}) + \frac{1}{2}
\]

58. \( y = 2 \cos(2\pi x - \pi) + \frac{1}{2} \)

- Amplitude: 2
- Period: \( \frac{2\pi}{2\pi} = 1 \)
- Phase Shift: \( \pi \times \frac{1}{2} \)
- Vertical Shift: Up \( \frac{1}{2} \)

59. \( y = -\tan \theta - 2 \)

- Amplitude: Reflected
- Period:
- Phase Shift:
- Vertical Shift: Down \( \frac{2}{2} \)
60. \( y = 3 \cot(\theta + \frac{\pi}{2}) \)
   Amplitude: 3
   Period: \( 2\pi \)
   Phase Shift: \( \frac{\pi}{2} \)
   Vertical Shift: -3

*Find the EXACT value of each expression, if it is defined.*

61. \( \sin^{-1} \left( \frac{1}{2} \right) \)

62. \( \cos^{-1} \left( -\frac{\sqrt{3}}{2} \right) \)

63. \( \tan^{-1} 0 \)

64. \( \sin^{-1} (0) \)

65. \( \cos^{-1} (1) \)

66. \( \sec^{-1} \left( \frac{2\sqrt{3}}{3} \right) \)

67. \( \csc^{-1} \sqrt{2} \)

68. \( \cot^{-1} \left( -\frac{\sqrt{3}}{2} \right) \)

69. \( \csc^{-1} (1) \)

70. \( \tan \left( \sin^{-1} \left( \frac{\sqrt{2}}{2} \right) \right) \)

71. \( \sin \left( \tan^{-1} \left( \frac{12}{5} \right) \right) \)

72. \( \csc \left( \cos^{-1} \left( -\frac{1}{2} \right) \right) \)

73. \( \sin^{-1} \left( \sin \left( \frac{\pi}{4} \right) \right) \)

74. \( \cos^{-1} \left( \cos \left( -\frac{\pi}{3} \right) \right) \)

75. \( \cot \left( \sin^{-1} \left( \frac{1}{2} \right) \right) \)

76. \( \sec \theta = \frac{1}{\cos \theta} \)

77. \( \frac{1 - \cos^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} \)

Simplify.

76. \( \frac{\sec \theta}{\cos \theta} = \frac{1}{\cos} \cdot \cos \theta \)

77. \( \frac{\tan^2 x}{\sin x \cos x} \)

\[ \tan x \]
78. \[ \frac{\sin 2x}{1-\cos 2x} = \frac{2\sin x \cos x}{1-(2\cos^2 x - 1)} \]

\[ \frac{2\sin x \cos x}{2-2\cos^2 x} = \frac{2\sin x \cos x}{2(1-\cos^2 x)} = \frac{2\sin x \cos x}{2 \sin^2 x} = \cot x \]

80. \[ \frac{\sin x + \cos x}{\tan x + \cot x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\tan x \cot x} \]

82. \[ \frac{\cos^2 x - \sin^2 x}{1-\tan^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x - \frac{\sin^2 x}{\cos^2 x}} = \frac{\cos^2 x}{\cos^2 x - \sin^2 x} = \cos^2 x \]

81. \[ \frac{\cos x \cos^2 x}{1-\sin x} + \frac{1-\sin^2 x}{\cos x} \]

\[ \frac{\cos^2 x + 1 - 2\sin x \sin^2 x}{\cos x(1-\sin x)} = \frac{2(1-\sin^2 x)}{\cos x(1-\sin x)} = 2 \sec x \]

83. \[ \frac{\tan^2 x}{\sec x} \]

\[ \frac{\sin^2 x \cdot \cos^2 x}{1} = \sin x \tan x \]

Use the sum and difference formulas to find the exact value.
84. \[ \cos 195^\circ \quad \cos(150^\circ + 45^\circ) \]

\[ \cos 150^\circ \cos 45^\circ - \sin 150^\circ \sin 45^\circ \]

\[ -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{\sqrt{6} - \sqrt{2}}{4} \]

85. \[ \sin 255^\circ \quad \sin(210^\circ + 45^\circ) \]

\[ \sin 210^\circ \cos 45^\circ + \cos 210^\circ \sin 45^\circ \]

Find \( \sin 2x, \cos 2x, \) and \( \tan 2x \) from the given information.
86. \[ \cos x = \frac{4}{5}, \quad \csc x < 0 \]

\[ \sin 2x = 2 \left( \frac{3}{5} \right) \left( \frac{4}{5} \right) = \frac{-24}{25} \]

\[ \cos 2x = 2 \left( \frac{4}{5} \right)^2 - 1 = \frac{7}{25} \]

\[ \tan 2x = -\frac{\frac{24}{25}}{\frac{7}{25}} = -\frac{24}{7} \]

Use the half-angle formulas to find the exact value.
88. \[ \sin 22.5^\circ \]

\[ \sin \left( \frac{45^\circ}{2} \right) = \frac{\sqrt{2 - \sqrt{2}}}{2} \]

\[ \cos 22.5^\circ = \frac{\sqrt{2 + \sqrt{2}}}{2} \]

\[ \tan \left( \frac{45^\circ}{2} \right) = 1 + \frac{\sqrt{2}}{2} \]

89. \[ \tan 67.5^\circ \]

\[ \frac{2 + \sqrt{2}}{2} \cdot \frac{1}{2} = \frac{2 + \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2} + 1}{2} \]

90. \[ \cos 112.5^\circ \]

\[ \frac{\sqrt{2 - \sqrt{2}}}{2} \cdot \frac{1}{2} = -\frac{1}{\sqrt{2} - 1} \]
Solve for $\alpha, 0^\circ \leq \alpha < 360^\circ$. Solve for $x, 0 \leq x < 2\pi$. Round your answer to the nearest thousandth.

91. $5 \sin^2 \alpha - 2 = 0$
\[
\sin^2 \alpha = \frac{2}{5}
\]
\[\alpha = 39.23^\circ, 140.76^\circ, 219.23^\circ, 320.76^\circ\]

92. $\tan^2 x - 9 = 0$
\[
\tan x = \pm 3
\]
\[X = 1.249, 1.893, 4.391, 5.039\]

Find the formulas giving the general solution for each given $0 \leq x < 2\pi$ and $0^\circ \leq \alpha < 360^\circ$. Your answers must be exact.

93. $\sin 3x + 1 = 0$
\[
3x = -\frac{\pi}{2} + 2\pi k
\]
\[x = -\frac{\pi}{6} + \frac{2\pi}{3} k
\]

95. $\csc^2 x - 4 = 0$
\[
\csc x = \pm \frac{2}{2}
\]
\[x = \frac{\pi}{6} + \pi k, \frac{5\pi}{6} + \pi k
\]

97. $2 \cos 2x = \sqrt{3}$
\[
2x = 30^\circ + 360^\circ k
\]
\[x = 15^\circ + 180^\circ k, 165^\circ + 180^\circ k
\]

99. $4 \sin^2 \theta = 3$
\[
\sin^2 \theta = \frac{3}{4}
\]
\[\sin \theta = \pm \frac{\sqrt{3}}{2}
\]
\[\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}
\]

100. $2 \cos^2 \theta = 1$
\[
\cos^2 \theta = \frac{1}{2}
\]
\[\cos \theta = \pm \frac{\sqrt{2}}{2}
\]
\[\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}
\]

Find all solutions of the equation in the interval $(0, 2\pi)$.

101. $\sin 2x = -\sin x$
\[
2 \sin x \cos x + \sin x = 0
\]
\[\sin x (2 \cos x + 1) = 0
\]
\[\sin x = 0, \cos x = -\frac{1}{2}
\]
\[x = 0\pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}
\]

102. $\sin x \tan x = 0$
\[
\sin x \cdot \frac{\sin x}{\cos x} = 0
\]
\[\sin^2 x = 0
\]
\[\sin x = 0
\]
\[x = 0\pi, \pi
\]