Inter section  \( A \cap B \)  \( A \text{ AND } B \)  

Union  \( A \cup B \)  \( A \text{ OR } B \)

Overlap Only!

Probability of A OR B (Union):

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

Mutually Exclusive:

If the intersection is empty and thus \( P(A \cup B) = P(A) + P(B) \)

Probability of A AND B (Intersection):

\[ P(A \cap B) = P(A) \cdot P(B) \]

Independent vs. Dependent:

Independent only if \( P(A \cap B) = P(A) \cdot P(B) \) Occurrence of one has no impact on occurrence of the other.

EXAMPLE 1 – Finding Probabilities of Compound Events

A single card is randomly selected from a standard deck of 52 cards. What is the probability that it is a:

a. ace or an eight

\[ \frac{4}{52} + \frac{4}{52} = \frac{2}{13} \]

c. face card or a spade

\[ \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{11}{26} \]

d. a red card or a 9

\[ \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{7}{13} \]

EXAMPLE 2 – Applying the Formula

a. Out of 200 students in a senior class, 113 are either varsity athletes or on the honor roll. There are 74 seniors who are varsity athletes and 51 seniors who are on the honor roll. What is the probability that a randomly selected senior is both a varsity athlete and on the honor roll?

\[ \frac{113}{200} = \frac{74}{200} + \frac{51}{200} - \frac{Both}{200} \]

Both = \( \frac{124}{200} = \frac{3}{50} \)

b. Of 100 students surveyed, 92 own either a car or a computer. Also, 65 own cars and 82 own computers. What is the probability that a randomly selected student owns both a car and a computer?

\[ \frac{92}{100} = \frac{65}{100} + \frac{82}{100} - \frac{Both}{100} \]

Both = \( \frac{55}{100} = \frac{11}{20} \)

Complements

The event that \( A \) Does Not Occur

Probability of the Complement of an Event:

\[ P(\overline{A}) = 1 - P(A) \]
EXAMPLE 3 – Finding Probabilities of Complements
When two six-sided dice are rolled, there are 36 possible outcomes, as shown. Find the probability of the given event.

a. The sum is not 5
\[ P(S \neq 5) = 1 - P(S = 5) = 1 - \frac{4}{36} = \frac{8}{9} \]
b. The sum is less than or equal to 8.
\[ P(S \leq 8) = 1 - P(S > 8) = 1 - \frac{13}{36} = \frac{13}{18} \]

EXAMPLE 4 – Using a Complement in Real Life
a. There are 10 people at a dinner party. What is the probability that at least 2 people have the same birthday?
\[ P(2 \text{ or more}) = 1 - P(\text{None}) = 1 - \frac{365 \cdot 364 \cdot 363 \ldots}{365^{10}} = 1 - \frac{365 \cdot 10}{365^{10}} = \frac{1169}{1165} \]
b. A restaurant gives a free fortune cookie to every guest. The restaurant claims there are 100 different messages hidden inside the fortune cookies. What is the probability that a group of 5 people receive at least 2 fortune cookies with the same message inside?
\[ P(2 \text{ or more}) = 1 - P(\text{None}) = 1 - \frac{100 \cdot 99 \cdot \ldots \cdot 96}{100^5} = 1 - \frac{10 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{100^5} = \frac{0.0965}{100} \]

EXAMPLE 5 – Finding Probabilities of Independent Events
a. In a survey at a football game, 50 of 75 male fans and 40 of 50 female fans said that they favor the new team mascot. If 1 male and 1 female fan are randomly selected, what is the probability that both favor the new mascot.
\[ P(M \cap F) = \frac{50}{75} \cdot \frac{40}{50} = \frac{8}{13} \]
b. For a fundraiser, a class sells 150 raffle tickets for a mall gift certificate and 200 raffle tickets for a booklet of movie passes. You buy 5 raffle tickets for each prize. What is the probability that you win the mall gift certificate but not the booklet of movie passes.
\[ P(\text{M} \cap \neg \text{L}) = \frac{5}{150} \cdot \frac{195}{200} = \frac{13}{100} \]
c. A survey found that 46% of parents surveyed say that they read to their children at least once a week. If 3 parents are selected at random, what is the probability that all 3 will say that they read to their children at least once a week?
\[ P(\text{All 3}) = .46 \cdot .46 \cdot .46 = .097 = \frac{97}{1000} \]
d. A spinner is divided into ten equal regions numbered 1 to 10. What is the probability that 3 consecutive spins result in perfect squares.
\[ P(1, 4, 9) = \frac{3}{10} \cdot \frac{3}{10} \cdot \frac{3}{10} = \frac{27}{1000} \]
e. What is the probability that at least one of the spins is a perfect square?
\[ P(\text{at least 1}) = 1 - P(\text{None}) = 1 - \frac{7}{10} \cdot \frac{7}{10} \cdot \frac{7}{10} = \frac{657}{1000} \]
EXAMPLE 6 – Comparing Independent and Dependent Events
You randomly select two cards from a standard deck of 52 cards. What is the probability that the first card is a heart and the second is a club if:

a. You replace the first card before selecting the second.

\[ P(\heartsuit \cap \clubsuit) = \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{16} \]

b. You do not replace the first card.

\[ = \frac{13}{52} \cdot \frac{13}{51} = \frac{13}{204} \]

What is the probability that the first card is not a heart and the second is a heart if:

c. You replace the first card before selecting the second.

\[ P(\spadesuit \cap \heartsuit) = \frac{39}{52} \cdot \frac{13}{52} = \frac{3}{16} \]

d. You do not replace the first card.

\[ = \frac{39}{52} \cdot \frac{13}{51} = \frac{13}{68} \]

EXAMPLE 4 – Using Combinations in Probabilities
You have a bag of Power Ranger trading cards. 7 of them are the pink ranger, 4 the red ranger, 8 the green ranger, and 2 the white ranger. You reach into the bag and take 3 cards out at once.

a. What is the probability that you grab 2 pink rangers and one green ranger?

\[ \frac{\binom{7}{2} \cdot \binom{8}{1}}{\binom{21}{3}} = \frac{21 \cdot 8}{1330} = \frac{12}{95} \]

b. What is the probability that you grab two white rangers?

\[ \frac{\binom{2}{2} \cdot \binom{19}{1}}{\binom{21}{3}} = \frac{1 \cdot 19}{1330} = \frac{1}{70} \]

c. What is the probability that you grab 3 red rangers?

\[ \frac{\binom{4}{3}}{\binom{21}{3}} = \frac{4}{1330} = \frac{2}{665} \]