Let \( f(x) = x^2 \) and \( g(x) = \sqrt{x - 5} \) and \( h(x) = \frac{6-x}{3} \). Perform the indicated operation and state the domain when necessary.

1. \( f(g(6)) \)
2. \( g\left(\frac{1}{2}\right) \)
3. \( (f(6)) \)
4. \( f(g(x)) \)
5. \( f\left(\frac{1}{2}\right) \)
6. \( g\left(\frac{1}{2}\right) \)
7. \( g(f(x)) \)
8. \( (f^{-1}(x)) \)
9. \( (g(x)) \)

Without Graphing, determine whether or not the following functions have inverse functions.

10. \( f(x) = x^2 + 17 \)
11. \( f(x) = 2x + 18 + \pi \)
12. \( f(x) = 5x^4 + 17x \)

Find the following inverse functions, if they exist.

13. \( f(x) = \frac{x^4}{8} + 7 \)
14. \( f(x) = \frac{3}{5}x + 8 \)
15. \( f(x) = 2(x + 5)^\frac{3}{2} \)
Find the inverse function for each of the following.
16. \( f(x) = \log_3 x + 7 \)  
17. \( g(x) = e^{x-4} \)  
18. \( (x) = \log_6(x + 5) \)

Evaluate each of the following logarithms.
19. \( \log_9 27 \)  
20. \( \log_3 \sqrt{27} \)  
21. \( \log_8(4\sqrt{32})^{3x} \)  
22. \( \log_{10,0,0} \frac{1}{10,0,0} \)

Solve each of the following equations, if a solution exists.
23. \( \log_8 x = \frac{5}{2} \)  
24. \( \log_9 x = \frac{3}{2} \)  
25. \( \log_x 27 = \frac{3}{2} \)  
26. \( \log_x 125 = \frac{1}{2} \)

Condense the following logarithmic expressions into a single logarithm.
27. \( 4 \cdot 2 \log_6 a \)  
28. \( 2 \log_3 m \cdot \frac{1}{2} \log_3 n \cdot 3 \cdot \log_3 2 \)

Simplify the following logarithmic expressions.
29. \( \log_5 \frac{1}{250} + 3 + \log_5 2 \)  
30. \( \frac{1}{6}(2 \log_8 4 + 2 \log_8 2) \)
Solve the following logarithmic equations. Check for extraneous solutions. Round answers to the nearest thousandth.

31. \(3 + 2 \ln x = 10\)

32. \(\log_4 (3x) = \log_4 3 + \log_4 x\)

33. \(\log_4 x \quad \log_4 (x - 1) = \frac{1}{2}\)

34. \(\log_6 (2x) \quad 5 \quad \log_6 (7x + 10) = 1\)

35. \(\log (10x) \quad \log (2 + \sqrt{x}) = 1\)

36. \(\ln (x - 1) + \ln (x + 2) = 1\)

Solve the following exponential equations. You must have an exact answer.

37. \(25^{2x} = \frac{1}{125}\) \(25^{x-1}\)

38. \(81^{3-x} = \left(\frac{1}{9}\right)^{5x-6} \sqrt{27^x}\)

Solve the following exponential equations. Round answers to the nearest thousandth.

39. \(e^{2-3x} = 12\)

40. \(4e^{2x} = 7\)

41. \(12^x = 5^{x+4}\)
42. $4 + 3^x = 8$

43. $\frac{50}{1+e^{-x}} = 4$

44. $100(1.04)^{2x} = 300$

45. $x^2 2^x - 2^x = 0$

46. $4xe^{-3x} - 3xe^{-3x} = 0$

47. $e^{4x} + 4e^{2x} - 21 = 0$

48. A man invests $5,000 in an account that pays 8.5% interest per year, compounded quarterly on July 1, 2008.
   a. Find the amount after 3 years.
   b. During which month of what year will the amount double?

49. A man invests $6,500 in an account that pays 6% interest per year, compounded continuously.
   a. What is the amount after 2 years?
   b. How long will it take for the amount to be $8,000?

50. During which month of what year will it take for an investment of $1,000 deposited on February 1, 1998 to double in value if the interest rate is 8.5% per year, compounded continuously?
51. A sum of $1,000 was invested for 4 years, and the interest was compounded semiannually. If this sum amounted to $1,435.77 in the given time, what was the interest rate?

52. A culture contains 1,500 bacteria initially and doubles every 30 minutes.
   a. Find a function that models the number of bacteria at time $t$.
   b. Find the number of bacteria after two hours.
   c. After how many minutes will there be 4,000 bacteria?

53. Radium-226 has a half-life of 1,600 years. Suppose a sample of this substance has a mass of 22 mg.
   a. Find a function that models the amount of the sample remaining at time $t$.
   b. Find the mass remaining after 4,000 years.
   c. How long will it take for the sample to decay to a mass of 18 mg?

54. Cesium-137 has a half-life of 30 years. Suppose a sample of this substance has a mass of 10 g.
   a. Find a function that models the amount of the sample remaining at time $t$.
   b. Find the mass remaining after 80 years.
   c. How long will it take for the sample to decay to a mass of 2 g?
Without using a calculator, graph the exponential equation \( f(x) = 2^x \). Then, graph each of the transformed functions. Be sure to list all transformations in the order in which they must be graphed. Then, find the Domain and Range.

56. \( g(x) = 2^x + 4 \)

\[
\begin{array}{c|c}
\text{Domain:} & \text{Range:} \\
\hline
\end{array}
\]

57. \( (x) = 2 \cdot 2^{x+5} \)

\[
\begin{array}{c|c}
\text{Domain:} & \text{Range:} \\
\hline
\end{array}
\]

Without using a calculator, graph the exponential equation \( f(x) = \left(\frac{1}{2}\right)^x \). Then, graph each of the transformed functions. Be sure to list all transformations in the order in which they must be graphed. Then, find the Domain and Range.

58. \( g(x) = \left(\frac{1}{2}\right)^{x-3} \)

\[
\begin{array}{c|c}
\text{Domain:} & \text{Range:} \\
\hline
\end{array}
\]

59. \( (x) = 3 \left(\frac{1}{2}\right)^x \cdot 6 \)

\[
\begin{array}{c|c}
\text{Domain:} & \text{Range:} \\
\hline
\end{array}
\]

Using a Graphing Calculator, graph the following exponential equations. Then, find the Domain and Range.

60. \( f(x) = 2e^{x-5} + 1 \)

\[
\begin{array}{c|c}
\text{Domain:} & \text{Range:} \\
\hline
\end{array}
\]

Without Graphing, identify the Domain and Range of each of the following functions. Simple sketches may help!!!!

61. \( f(x) = 3^{x-9} + 7 \)

62. \( f(x) = 3 \cdot 4^x \cdot 6 \)
Without using a calculator, graph the logarithmic function $f(x) = \log_4 x$. Then, graph each of the transformed functions. Be sure to list all transformations in the order in which they must be graphed. Then, find the Domain and Range.

63. $g(x) = 2\log_4(x - 4)$

64. $(x) = \log_4 x + 4$

65. $f(x) = \log_{\frac{1}{2}}(x + 6)$

66. $f(x) = 3 \log_{\frac{1}{2}} x - 5$

Using a Graphing Calculator, graph the following exponential equations. Then, find the Domain and Range.

67. $f(x) = 3\ln(x - 6)$
Without Graphing, identify the Domain and Range of each of the following functions.
Simple sketches may help!!!!

68. \( f(x) = 14\log_8(x + 9) \)
69. \( f(x) = \log_{12}(x + 19) \)

70. \( f(x) = \log_7 x \)
71. \( f(x) = 3\log_5(x + 8) + 2 \)

Write an exponential function \( y = ab^x \) whose graph passes through the given points.

72. (1, 4) and (2, 16)
73. (1, 6) and (4, 162)
ANSWERS

1. \( f(g(6)) = 1 \)  \hspace{1cm} 2. \( g^{-1}(3) = \sqrt{10} \)  \hspace{1cm} 3. \( f(6) = 10 \)  \hspace{1cm} 4. \( f(x) = x \)  \hspace{1cm} 5. \( D: [5, \infty) \)

5. \( f^{-1}(x) = 9x^2 - 36x + 36; \ D: (\infty, \infty) \)  \hspace{1cm} 6. \( g(x) = \sqrt[3]{9-x}; \ D: (\infty, 9] \)

7. \( g(f(x)) = \sqrt{x^2 - 5}; \ D: (\infty, \sqrt{5}] \cup [\sqrt{5}, \infty) \)  \hspace{1cm} 8. \( f^{-1}(x) = \frac{6 \pm \sqrt{x}}{3}; \ D: [0, \infty) \)

9. \( (g(x)) = \frac{6 - \sqrt[3]{x} - 5}{3}; \ D: [5, \infty) \)  \hspace{1cm} 10. No  \hspace{1cm} 11. Yes  \hspace{1cm} 12. No  \hspace{1cm} 13. Does Not Exist

14. \( f^{-1}(x) = \frac{5}{3}x^3 \)  \hspace{1cm} 15. \( f^{-1}(x) = \frac{3 \sqrt[4]{x^2}}{5} \)  \hspace{1cm} 16. \( f^{-1}(x) = 3^{x-7} \)  \hspace{1cm} 17. \( g^{-1}(x) = \ln x + 4 \)

18. \( \sqrt[3]{x} = 5 \)  \hspace{1cm} 19. \( x = \frac{3}{2} \)  \hspace{1cm} 20. \( x = \frac{4}{3} \)  \hspace{1cm} 21. \( \frac{13x}{4} \)  \hspace{1cm} 22. 4  \hspace{1cm} 23. \( x = 2^\frac{7}{2} \)

24. \( x = \frac{1}{27} \)  \hspace{1cm} 25. \( x = 9 \)  \hspace{1cm} 26. \( x = \frac{1}{5^6} \)  \hspace{1cm} 27. \( \log_6 \frac{64}{a^2} \)  \hspace{1cm} 28. \( \log_3 \frac{m^2}{54n^5} \)  \hspace{1cm} 29. 0  \hspace{1cm} 30. \( x = \frac{1}{3} \)

31. \( x = 33.115 \)  \hspace{1cm} 32. \( (0, \infty) \)  \hspace{1cm} 33. \( x = 2 \)  \hspace{1cm} 34. \( x = 8 \)  \hspace{1cm} 35. \( x = 4 \)  \hspace{1cm} 36. \( x = 1.729 \)  \hspace{1cm} 37. \( x = \frac{5}{2} \)  \hspace{1cm} 38. \( x = 0 \)

39. \( x = 0.162 \)  \hspace{1cm} 40. \( x = 0.280 \)  \hspace{1cm} 41. \( x = 7.353 \)  \hspace{1cm} 42. \( x = 0.252 \)

43. \( x = 2.442 \)  \hspace{1cm} 44. \( x = 14.006 \)  \hspace{1cm} 45. \( x = \pm 1 \)  \hspace{1cm} 46. \( x = 0, \frac{4}{3} \)

47. \( x = 0.549 \)  \hspace{1cm} 48a. \( \$6,435.09 \)  \hspace{1cm} 48b. \( \text{September of 2016} \)  \hspace{1cm} 49a. \( \$7,328.73 \)

49b. 3.46 years  \hspace{1cm} 50. \( \text{Marc of 2006} \)  \hspace{1cm} 51. \( r \approx 9.25\% \)  \hspace{1cm} 52a. \( N = 1500(2)^{\frac{t}{50}} \)

52b. 24,000 bacteria  \hspace{1cm} 52c. \( t \approx 42.5 \text{ min} \)  \hspace{1cm} 53a. \( N = 22 \left(\frac{1}{2}\right)^{\frac{t}{160}} \)  \hspace{1cm} 53b. 3.89 mg

53c. 463.21 years  \hspace{1cm} 54a. \( N = 10 \left(\frac{1}{2}\right)^{\frac{t}{50}} \)  \hspace{1cm} 54b. 1.57 g  \hspace{1cm} 54c. 69.7 years

56. \( D: (\infty, \infty) \)  \hspace{1cm} 57. \( D: (\infty, \infty) \)  \hspace{1cm} 58. \( D: (\infty, \infty) \)

59. \( D: (\infty, \infty) \)  \hspace{1cm} 60. \( D: (\infty, \infty) \)  \hspace{1cm} 61. \( D: (\infty, \infty) \)

62. \( D: (\infty, \infty) \)  \hspace{1cm} 63. \( D: (4, \infty) \)  \hspace{1cm} 64. \( D: (0, \infty) \)

65. \( D: (6, \infty) \)  \hspace{1cm} 66. \( D: (0, \infty) \)  \hspace{1cm} 67. \( D: (6, \infty) \)

68. \( D: (9, \infty) \)  \hspace{1cm} 69. \( D: (19, \infty) \)  \hspace{1cm} 70. \( D: (0, \infty) \)

71. \( D: (8, \infty) \)  \hspace{1cm} 72. \( y = 1(4)^x \)  \hspace{1cm} 73. \( y = 2(3)^x \)