### Parabolas

<table>
<thead>
<tr>
<th>Equation</th>
<th>Direction of Opening</th>
<th>Vertex</th>
<th>Focus</th>
<th>Directrix</th>
<th>Focal Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = \frac{1}{4p} y^2$</td>
<td>right if $p$ is +</td>
<td>(0,0)</td>
<td>(p,0)</td>
<td>$x = -p$</td>
<td>$FD = 4p$</td>
</tr>
<tr>
<td>$y = \frac{1}{4p} x^2$</td>
<td>up if $p$ is +</td>
<td>(0,0)</td>
<td>(0,p)</td>
<td>$y = -p$</td>
<td>Line connecting points on either side of focus.</td>
</tr>
<tr>
<td>$(x - h) = \frac{1}{4p} (y - k)^2$</td>
<td>down if $p$ is +</td>
<td>(h,k)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(y - k) = \frac{1}{4p} (x - h)^2$</td>
<td></td>
<td>(h,k)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 1 – Identifying Information**

- **a.** Focus: (4, -3); Directrix: $y = 1$; Find Vertex
  - Vertex: $(4, -1)$
  - Focus: $(0, 0)$

- **b.** Focus: (2, 0); Vertex: (2, 2); Find Directrix
  - Directrix: $x = -1$; Find Focus

**Example 2 – Graphing Equations of Parabolas**

Find an equation and graph the parabola. Identify the vertex, focus, directrix, axis of symmetry, and domain and range.

- **a.** Vertex $(0, 0)$ and Focus $(0, -2)$
  - Vertex: $(0, 0)$
  - Focus: $(0, -2)$
  - Directrix: $y = 2$
  - Axis of Symmetry: $x = 0$
  - Equation: $y = \frac{1}{8} x^2$

- **b.** Focus $(1, 0)$ and Directrix $y = 4$
  - Vertex: $(1, 0)$
  - Focus: $(1, 0)$
  - Directrix: $y = 4$
  - Axis of Symmetry: $x = 0$
  - Equation: $y = \frac{1}{8} x^2$
c. Focus (0, 0) and Directrix \( x = -6 \)
   
   Vertex: \((-3, 0)\)
   
   Focus: \((0, 0)\)
   
   Directrix: \(x = -6\)
   
   Axis of Symmetry: \(y = 0\)
   
   Equation: \(\frac{(x-(-3))^2}{4+3} = \frac{1}{12} y^2\)
   
   \(FD = 4\) \((3) = 12\)

**EXAMPLE 3 - Rewriting Equations**

Find an equation and graph the parabola. Identify the vertex, focus, directrix, axis of symmetry, and domain and range.

a. \(4y + x^2 = 0\)
   
   \(4y = -x^2\)
   
   \(y = \frac{1}{4} x^2\)
   
   \(\frac{1}{4} = \frac{1}{4}p\)
   
   \(p = -1\)
   
   Vertex: \((0, 0)\)
   
   Focus: \((0, -1)\)
   
   Directrix: \(y = 1\)
   
   Axis of Symmetry: \(x = 0\)
   
   Equation: \(y = -\frac{1}{4} x^2\)
   
   \(FD = 4\)

b. \((y - 1)^2 = -8x\)
   
   \(y = x^2 - 6x - 4y + 5 = 0\)
   
   \(x^2 - 6x + \frac{9}{4} = 4y - 5 + \frac{9}{4}\)
   
   \((x - 3)^2 = 4y - \frac{9}{4}\)
   
   \(\frac{1}{4} (x - 3)^2 = y + 1\)
   
   \(p = 1\)
   
   Vertex: \((3, -1)\)
   
   Focus: \((3, 0)\)
   
   Directrix: \(y = -2\)
   
   Axis of Symmetry: \(x = 3\)
   
   Equation: \(y + 1 = \frac{1}{4} (x-3)^2\)
   
   \(D: (-\infty, \infty)\) \(R: [-1, \infty)\)

d. \(4x = y^2 + 4y\)
   
   \((x-3)^2 = 4(y+1)\)
   
   \(p = 1\)
   
   Vertex: \((3, 1)\)
   
   Focus: \((3, 0)\)
   
   Directrix: \(y = 2\)
   
   Axis of Symmetry: \(x = 3\)
   
   Equation: \(y+1 = \frac{1}{4} (x-3)^2\)
   
   \(D: (-\infty, \infty)\) \(R: [-1, \infty)\)