Sketch the graph of the quadratic function. Identify the vertex, x-intercept(s) and y-intercept.

1. \( f(x) = -x^2 - 2 \)
   - Vertex: \((0, -2)\)
   - X-Intercept(s): none
   - Y-Intercept: \((0, -2)\)

2. \( f(x) = x^2 - 6x + 9 \)
   - Vertex: \((3, 0)\)
   - X-Intercept(s): \((3, 0)\)
   - Y-Intercept: \((0, 9)\)

3. \( f(x) = x^2 - 2x - 8 \)
   - Vertex: \((1, -9)\)
   - X-Intercept(s): \((4, 0); (-2, 0)\)
   - Y-Intercept: \((0, -8)\)

Find a function whose graph is a parabola with the given vertex and that passes through the given point. Leave your answer in standard form \( y = a(x - h)^2 + k \)

4. Vertex: \((1, -2)\); Point: \((4, 16)\)
   \[ 16 = a(4 - 1)^2 - 2 \]
   \[ 16 = 9a - 2 \]
   \[ 18 = 9a \]
   \[ a = 2 \]
   \[ y = 2(x - 1)^2 - 2 \]

5. Vertex: \((3, 4)\); Point: \((1, -8)\)
   \[ -8 = a(1 - 3)^2 + 4 \]
   \[ -8 = 4a + 4 \]
   \[ -12 = 4a \]
   \[ a = -3 \]
   \[ y = -3(x - 3)^2 + 4 \]

6. A ball is thrown across a playing field. Its path is given by the equation \( y = -0.005x^2 + x + 5 \), where \( x \) is the distance the ball has traveled horizontally, and \( y \) is its height above ground level, both measured in feet.

   a. What is the maximum height attained by the ball?
   \[ y \text{ coordinate of the vertex} \]
   \[ \frac{-1}{2(-0.005)} = 100 \]
   \[ 0.005(100)^2 + 100 + 5 = 55 \]

   6a.) 55 ft.
b. How far has it traveled horizontally when it hits the ground? 
\[ x = \text{intercept} \]
\[ 0 = -0.005x^2 + x + 5 \]

\[ \text{use graphing calculator or quadratic formula} \]

7. The effectiveness of a television commercial depends on how many times a viewer watches it. After some experiments an advertising agency found that if the effectiveness \( E \) is measured on a scale of 0 to 10, then \( E(n) = \frac{2}{3}n - \frac{1}{90}n^2 \) where \( n \) is the number of times a viewer watches a given commercial. For a commercial to have maximum effectiveness, how many times should a viewer watch it?

\[ \text{x-coordinate of the vertex} \]
\[ x = \frac{-\frac{2}{3}}{2(-\frac{1}{90})} = 30 \]

\[ \text{graph opens down, makes sense to maximize} \]
\[ 7.) \ 30 \text{ times} \]

8. Find two positive numbers whose sum is 100 and the sum of whose squares is a minimum.

\[ \#1 = x \quad f(x) = x^2 + (100 - x)^2 \]
\[ \#2 = 100 - x \quad f(x) = x^2 + 10000 - 200x + x^2 \]

\[ = 2x^2 - 200x + 10000 \]

\[ x = \frac{200}{2(2)} = \frac{200}{4} = 50 \]

\[ \text{x-coordinate of vertex} \]

9. A farmer has 2400 feet of fencing and wants to fence off a rectangular field that borders a straight river. He does not need a fence along the river.

\[ \text{graph opens down, makes sense to maximize} \]

a. Find a function that models the area of the field in terms of one of its sides.

\[ A(x) = x(2400 - 2x) = -2x^2 + 2400x \]

\[ 9a.) A(x) = -2x^2 + 2400x \]

b. What are the dimensions of the field that would produce the maximum area?

\[ \text{x-coordinate of the vertex} \]
\[ x = \frac{-2400}{2(-2)} = 600 \]

\[ 9b.) \ 600 \text{ ft by 1200 ft} \]

9c.) \[ 720,000 \text{ ft}^2 \]

10. Find all the real zeros of the polynomial function. Determine the multiplicity of each zero. Use the leading coefficient test and end behavior to sketch the graph.

\[ f(x) = -x^4 + 8x^3 - 15x^2 \]

\[ = -x^2(x^2 - 8x + 15) \]

\[ = -x^2(x - 3)(x - 5) \]

10. Zeros:

\[ \begin{align*}
0 \text{ mult. 2} \\
3 \text{ mult. 1} \\
5 \text{ mult. 1}
\end{align*} \]

End Behavior:

\[ x \to \infty \quad f(x) \to -\infty \]

\[ x \to -\infty \quad f(x) \to -\infty \]

\[ \text{degree is even} \]

\[ \text{L.C. is negative} \]
11. Find a polynomial function with the given zeros, multiplicities and degree.

12. Zero: $-4$, multiplicity 2  
Zero: $1 + \sqrt{2}$, multiplicity 1  
Zero: $1 - \sqrt{2}$, multiplicity 1  
Falls to the right, falls to the left
\[(x+4)^2(x-(1+\sqrt{2}))(x-(1-\sqrt{2}))\]  
\[(x^2+8x+16)(x-1-\sqrt{2})(x-1+\sqrt{2})\]  
\[(x^2+8x+16)((x-1)^2-(\sqrt{2})^2)\]  
\[(x^2+8x+16)(x^2-2x+1-2)\]

Find all rational zeros of the function. Identify the multiplicity of each zero.

13. $f(x) = x^3 - 4x^2 - 7x + 10$

$p: 10 \rightarrow 1, 2, 5, 10$

$q: 1 \rightarrow 1$

$p/q: \pm 1, \pm 2, \pm 5, \pm 10$

14. $f(x) = x^3 - x^2 - 8x + 12$

$p: 12 \rightarrow 1, 2, 3, 4, 6, 12$

$q: 1 \rightarrow 1$

$p/q: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
Find all rational zeros of the function. Identify the multiplicity of each zero. [continued]

15. \( f(x) = x^4 - 2x^3 - 3x^2 + 8x - 4 \)

\[
\begin{array}{r|rrrr}
& 1 & -2 & -3 & 8 & -4 \\
\hline
4 & 4 & -4 & 0 & 0 & 0 \\
\hline
1 & 1 & -1 & -4 & 0 & 0
\end{array}
\]

\* test 1 again right away! it might be a double root!!

\* when the degree of the polynomial is greater than three, always test for a double root... it might be the only other rational solution!

-2 \mult 1

16. \( f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8 \)

\[
\begin{array}{r|rrrr}
& 1 & -5 & 6 & 4 & -8 \\
\hline
8 & 8 & -4 & 2 & 6 & -2 \\
\hline
1 & 1 & -4 & 2 & 6 & -2
\end{array}
\]

\* No!

\[
\begin{array}{r|rrrr}
& 1 & -5 & 6 & 4 & -8 \\
\hline
2 & 2 & -8 & 8 & -8 & 0 \\
\hline
1 & -6 & 12 & -8 & 0
\end{array}
\]

test for double root, if not D.R. then move on to next possible rational zero

-2 \mult 1

17. Use the Remainder Theorem to find \( f(1) \) for \( f(x) = 4x^4 - 16x^3 + 7x^2 + 20 \).

\( f(1) = 4(1)^4 -16(1)^3 +7(1)^2 + 20 = 15 \)

18. Using synthetic division, find the remainder for \( f(x) \) in #17, if it is divided by \( (x - 1) \).

\[
\begin{array}{r|rrrr}
& 4 & -16 & 7 & 20 \\
\hline
1 & 4 & -12 & -5 & -5
\end{array}
\]

19. Compare your results from #17 and #18. Describe those results.

When the divisor is linear, \( x - c \), doing synthetic division produces both \( q(x) \) and \( r(x) \). The Remainder Theorem, evaluated at \( c \), finds just \( r(x) \).